# SIMC L/T/LT/TT parameterization

Garth Huber

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# Pre-requisite: Stable and finalized data!

- **Before starting the L/T/LT/TT-separation procedure, it is essential that you have:**
  - **Final normalized yields (counts/mC) for all settings, with all efficiencies, livetimes, cryotarget, FADC-DT and other yield corrections tested for reliability over a wide rate range and applied**
  - **All kinematic offsets determined and finalized**
- This is because it is essential that the one thing that is kept constant in the iterations is the experimental normalized yield and distributions.
- If any subsequent changes are made to any part of the experimental distributions, the iteration procedure must be repeated to ensure the result remains self-consistent.
- **Failure to respect this restriction will result in significant wasted time!**

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# Pre-requisite – Choose a functional form

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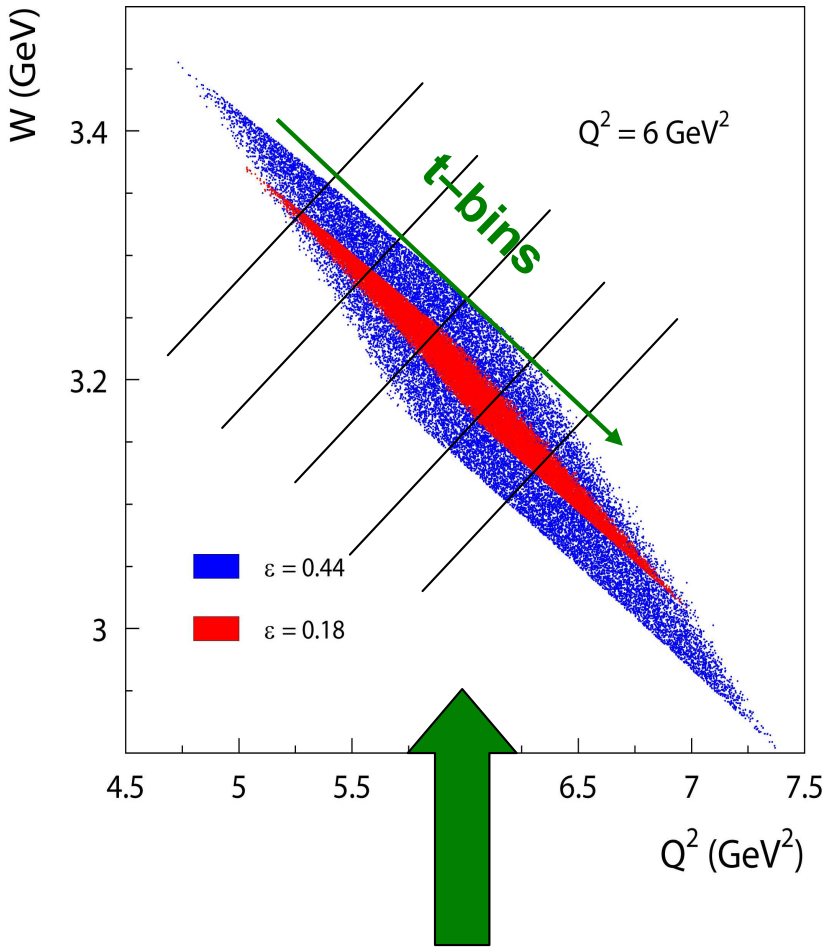
- **The cross section varies across experimental acceptance.**
- **It is needed to choose a functional form that will reasonably take into account this variation.**
- Of course you don't know in advance what to choose, therein the uncertainty.
- All you can do is to make a choice, and start the iteration process with it.
- Then you need to do tests to see if the functional dependence reproduces the variation of the data.
- If the tests fail, then you need to modify the functional form and try again until you get something that works better.




Each t bin has a different average value of W, $Q^2$. **This dependence must be taken into account**

# Example Parameterizations Used

- Fpi–1 LH+
- Blok et al., PRC **78** (2008) 045202

This procedure was carried out independently for Fpi-1 and Fpi-2 in order to have optimal descriptions in the two different kinematic ranges covered.[4] The final cross section parametrization for Fpi-1 (the cross sections have units of $\mu$b/GeV$^2$, and the units of $Q^2$, $t$, and $m_\pi^2$ are GeV$^2$) is

$$\frac{d\sigma_L}{dt} = 36.51 e^{(26.10-7.75Q^2)(t+0.02)},$$
$$\frac{d\sigma_T}{dt} = \frac{0.74}{Q^2} + \frac{1.25}{Q^4} + 0.57\frac{|t|}{\left(|t|+m_\pi^2\right)^2},$$
$$\frac{d\sigma_{LT}}{dt} = \left(\exp\left[4.69 + \frac{24.55}{\sqrt{Q^2}}t\right] + 1.47 - \frac{7.89}{Q^4}\right)\sin\theta^*,$$
$$\frac{d\sigma_{TT}}{dt} = \left(\frac{3.44}{Q^2} - \frac{7.57}{Q^4}\right)\frac{|t|}{\left(|t|+m_\pi^2\right)^2}\sin^2\theta^*. \quad (15)$$

This parametrization is valid in the $Q^2$ range between 0.4 and 1.8 GeV$^2$.

- Fpi–2 LH+
- Blok et al., PRC **78** (2008) 045202

The Fpi-2 parametrization, valid between $Q^2 = 1.4$ and 2.7 GeV$^2$, is

$$\frac{d\sigma_L}{dt} = \frac{350Q^2}{(1+1.77Q^2+0.05Q^4)^2}e^{(16-7.5\ln Q^2)t},$$
$$\frac{d\sigma_T}{dt} = \frac{4.5}{Q^2} + \frac{2.0}{Q^4}, \quad (16)$$
$$\frac{d\sigma_{LT}}{dt} = \left(\exp\left[0.79 + \frac{3.4}{\sqrt{Q^2}}t\right] + 1.1 - \frac{3.6}{Q^4}\right)\sin\theta^*,$$
$$\frac{d\sigma_{TT}}{dt} = -\frac{5.0}{Q^4}\frac{|t|}{\left(|t|+m_\pi^2\right)^2}\sin^2\theta^*.$$

It's okay to have different parameterizations for different Q², W ranges, if that's what's required to have a good fit

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# Example Parameterizations Used

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- Fpi–1 LD+
- Huber et al., PRC **91** (2015) 015202

This procedure was carried out independently for $\pi^+$ and $\pi^-$ at each $Q^2$, to have optimal descriptions in the different kinematic ranges covered. The parametrizations used in the $F_\pi$-1 $\pi^+$ analysis are

$$\frac{d\sigma_L}{dt} = g(W)[p_1 + p_2 \ln(Q^2)]e^{[p_3+p_4 \ln(Q^2)](-t)},$$
$$\frac{d\sigma_T}{dt} = g(W)\left(\frac{|t| - |t_{\rm ave}|}{|t_{\rm ave}|}\right)\{p_5 + p_6 \ln(Q^2) + [p_7 + p_8 \ln(Q^2)]\},$$
$$\frac{d\sigma_{LT}}{dt} = g(W)p_9 e^{p_{10}(-t)} \sin\theta_{\rm c.m.},$$
$$\frac{d\sigma_{TT}}{dt} = g(W)f(t)\frac{p_{11}}{Q^2}e^{-Q^2} \sin^2\theta_{\rm c.m.}, \quad (9)$$

where $g(W) = 1/(W^2 - m_p^2)^2$ is the assumed $W$ dependence discussed earlier, $f(t) = -t/(-t - m_\pi^2)^2$ is the pion pole factor, $|t_{\rm ave}|$ is the average $-t$ value for a given kinematic setting, given by $|t_{\rm ave}| = [0.105 + 0.04 \ln(Q^2)]Q^2$, and $p_{i=1,\ldots,12}$ are the fit parameters.

- Fpi–1 LD–
- Huber et al., PRC **91** (2008) 015202

For the $F_\pi$-1 $\pi^-$ analysis, a slightly different parametrization (because $\sigma_T$ and $\sigma_{TT}$ showed a stronger $Q^2$-dependence) yielded a better fit:

$$\frac{d\sigma_L}{dt} = g(W)[p_1 + p_2 \ln(Q^2)]e^{[p_3+p_4 \ln(Q^2)](-t)},$$
$$\frac{d\sigma_T}{dt} = g(W)\left\{p_5 + \frac{p_6}{Q^4 + 0.1} + [p_7 + p_8 \ln(Q^2)]\left(\frac{|t| - |t_{\rm ave}|}{|t_{\rm ave}|}\right)\right\},$$
$$\frac{d\sigma_{LT}}{dt} = g(W)p_9 e^{p_{10}(-t)} \sin\theta_{\rm c.m.},$$
$$\frac{d\sigma_{TT}}{dt} = g(W)f(t)\left(\frac{p_{11}}{Q^2} + \frac{p_{12}}{Q^4 + 0.2}\right)\sin^2\theta_{\rm c.m.}. \quad (10)$$

Pay attention to note above that LD– T, TT $Q^2$–dependences had to be modified to yield a better fit

# Example Parameterizations Used

- Fpi–2 LD+/–
- Huber et al., PRC **91** (2015) 015202

In the $F_\pi$-2 analyses, a common parametrization (similar to those in $F_\pi$-1) was used for both $\pi^+$ and $\pi^-$,

$$\frac{d\sigma_L}{dt} = g(W)[p_1 + p_2 \ln(Q^2)]e^{[p_3+p_4 \ln(Q^2)](-t-0.2)},$$

$$\frac{d\sigma_T}{dt} = g(W)\Big\{p_5 + p_6 \ln(Q^2) + [p_7 + p_8 \ln(Q^2)]\left(\frac{|t| - |t_{\rm ave}|}{|t_{\rm ave}|}\right)\Big\},$$

$$\frac{d\sigma_{LT}}{dt} = g(W)\left[p_9 e^{p_{10}(-t)} + \frac{p_{11}}{(-t)}\right]\sin\theta_{\rm c.m.},$$

$$\frac{d\sigma_{TT}}{dt} = g(W) f(t)\frac{p_{12}}{Q^2}e^{-Q^2}\sin^2\theta_{\rm c.m.}, \qquad (11)$$

where $|t_{\rm ave}| = [0.0735 + 0.028 \ln(Q^2)]Q^2$ and $p_4 = 0$.

LT and TT have extra sinθ* dependences since they are required to vanish in parallel–kinematics limit, i.e. θ*=0

The $|t|–|t_{ave}|$ factors vanish near the mean $t$ for each ($Q^2$,W) kinematics. We wanted to incorporate a $t$–dependence that would only be a small correction away from $t_{ave}$, to make the iterations converge more easily. You would need your own calculation of $t_{ave}$

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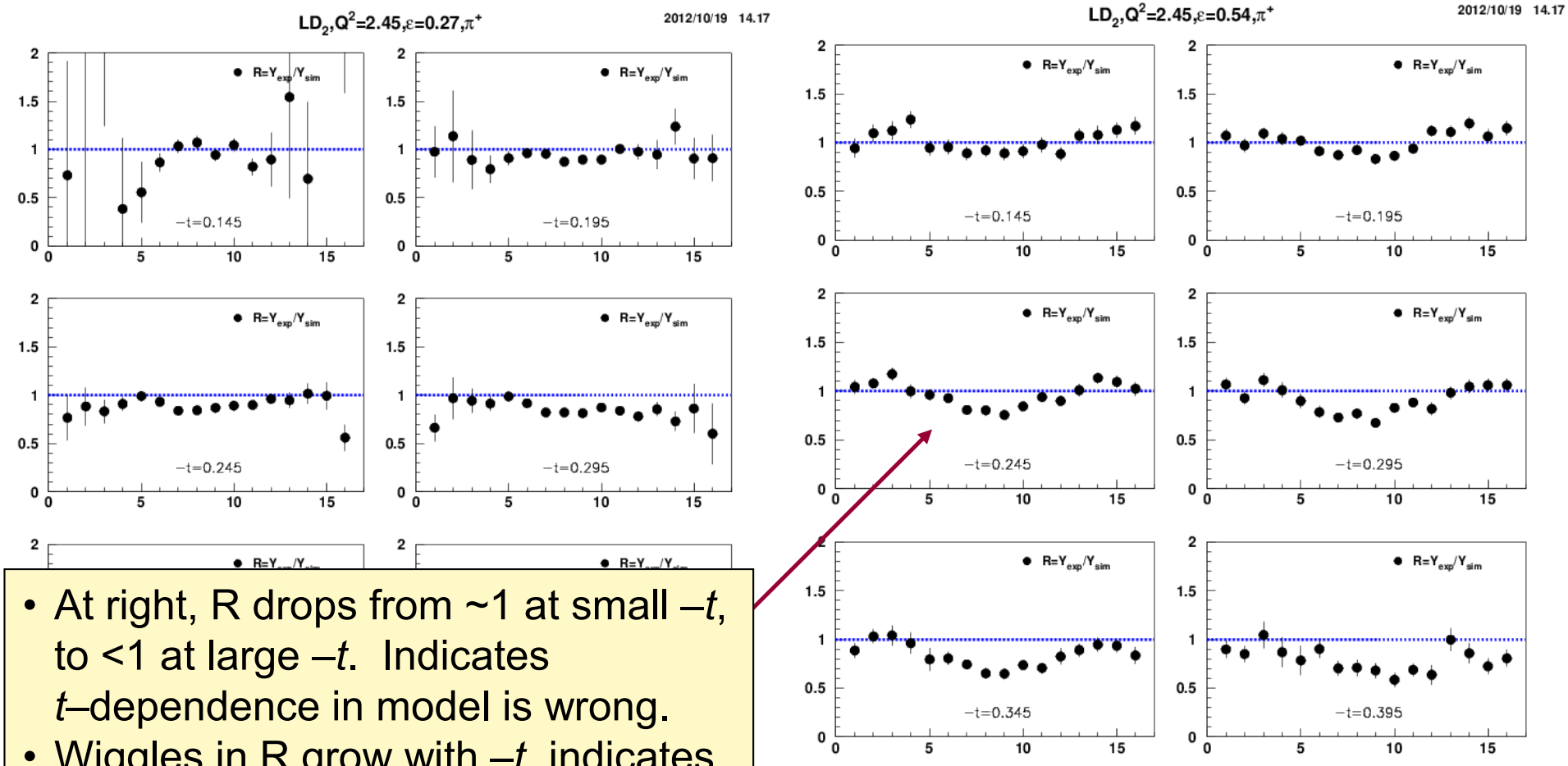
# Evaluating if Fit parameters are okay

- Deviations between Data and MC usually are indicated as wiggles in R. We desire R≈1 over broad kinematic range




- At right, R drops from ~1 at small –*t*, to <1 at large –*t*. Indicates *t*–dependence in model is wrong.
- Wiggles in R grow with –*t*, indicates problem with interference terms

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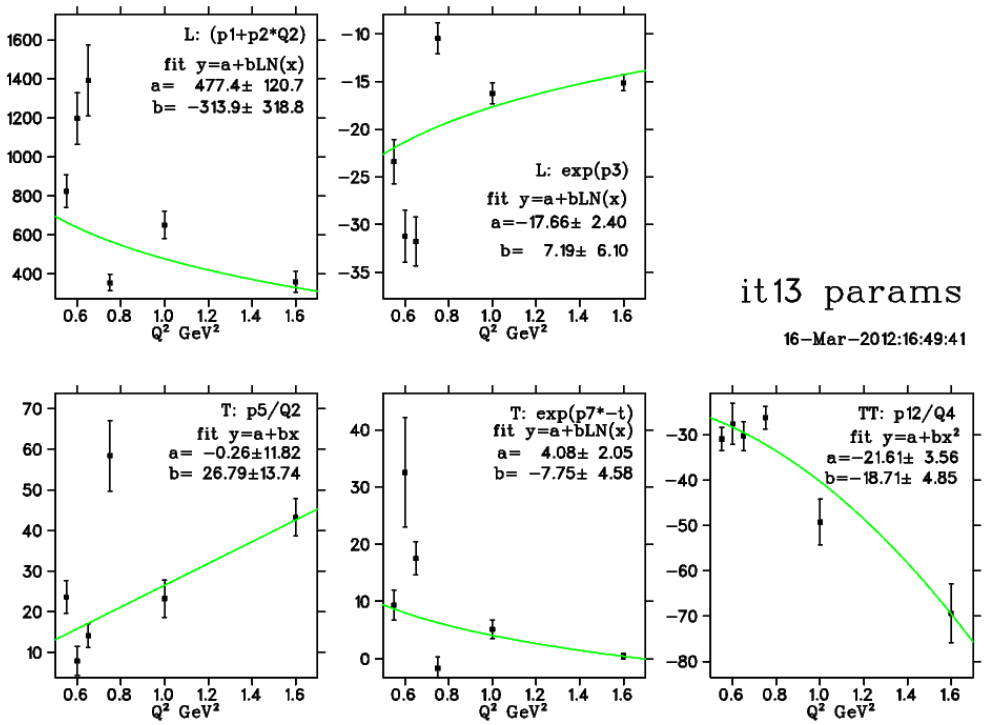
# Evaluating if Fit equations are okay

- Fit parameters are not required to be the same for different kinematics, but a consistent trend with $Q^2$ could indicate the parameterization equations need adjustment
- We compared fitpar() from different $Q^2$,W to see if they were slowly varying
- Here, $p5/Q^2$ parameter in T and $p12/Q^4$ parameter in TT grow with $Q^2$, indicating a steeper $Q^2$–dependence could be needed in model




**More details are found in LD2 analysis technical report HallC–docDB–773**

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# SIMC modifications

- Replace physics_pion.f or physics_kaon.f with physics_iterate.f in SIMC
- Change to SIMC Makefile:

```
## CEBAF DEFAULT SETUP FLAGS:
simcdir = .

## THE REST SHOULD BE OK WITHOUT MODIFICATION.

## This tells make not to delete these target files on error/interrupt (see man page)
.PRECIOUS: *.o sos/*.o hms/*.o hrsl/*.o hrsr/*.o shms/*.o calo/*.o

RM        = rm -f
SHELL     = /bin/sh
S       = $(simcdir)/sos/
H       = $(simcdir)/hms/
L       = $(simcdir)/hrsl/
R       = $(simcdir)/hrsr/
A       = $(simcdir)/shared/
SH      = $(simcdir)/shms/
T       = $(simcdir)/cteq5/
C       = $(simcdir)/calo/
CH      = $(simcdir)/cern/
D       = $(simcdir)/fdss/

OBJ1    = target.o brem.o gauss1.o NtupleInit.o NtupleClose.o enerloss_new.o
OBJ2    = radc.o init.o dbase.o physics_kaon.o physics_pion.o physics_delta.o physics_proton.o loren.o sf_lookup.o
OBJ3    = semi_physics.o rho_physics.o rho_decay.o generate_rho.o trg_track.o semi_dilution.o
OBJ4    = results_write.o event.o call_ranlux.o jacobians.o F1F2IN21.o
OBJ5    = $(A)musc.o $(A)musc_ext.o $(A)project.o $(A)transp.o
OBJ6    = $(A)rotate_haxis.o $(A)rotate_vaxis.o $(A)locforunt.o
OBJ7    = $(H)mc_hms.o $(H)mc_hms_hut.o $(H)mc_hms_recon.o
OBJ8    = $(S)mc_sos.o $(S)mc_sos_hut.o $(S)mc_sos_recon.o
```

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# SIMC modifications

- Replace physics_pion.f with physics_iterate.f in SIMC

libra:/home/huberg/r2d2/simc/simc_fpi2

File Edit View Search Terminal Help

```
* Models for sigL, sigT, sigLT, sigTT  for Deuterium.

***
* Parameterization revised for IT26, 12.11.09
          q2_set=2.45
          tav=(0.0735+0.028*log(q2_set))*q2_set
          ftav=(abs(t_gev)-tav)/tav
          ft=t_gev/(abs(t_gev)+0.139570**2)**2

          sigl=(fitpar(1)+fitpar(2)*log(Q2_g))
     1          *exp((fitpar(3)+fitpar(4)*log(Q2_g))*(abs(t_gev)-0.2))
          sigt=fitpar(5)+fitpar(6)*log(Q2_g)
     1          +(fitpar(7)+fitpar(8)*log(Q2_g))*ftav

          siglt=(fitpar(9)*exp(fitpar(10)*abs(t_gev))
     1          +fitpar(11)/abs(t_gev))*sin(thetacm)
          sigtt=(fitpar(12)*Q2_g*exp(-Q2_g))*ft*sin(thetacm)**2

          tav=(-0.178+0.315*log(Q2_g))*Q2_g



          sig219=(sigt+main%epsilon*sigl+main%epsilon*cos(2.*phicm)*sigtt
     >          +sqrt(2.0*main%epsilon*(1.+main%epsilon))*cos(phicm)*siglt)/1.d0

c now convert to different W
c W dependence given by 1/(W^2-M^2)^2
c factor 15.333 is value of (w**2-ami**2)**2 at W=2.19

c         wfactor=15.333/(s-(targ.Mtar_pion/1000.)**2)**2
*         wfactor=8.539/(s-(targ.Mtar_pion/1000.)**2)**2

c         wfactor=1.D0/(s-(targ.Mtar_pion/1000.)**2)**2
          wfactor=1.D0/(s_gev-mtar_gev**2)**2
          sig=sig219*wfactor
          sigl=sigl*wfactor
          sigt=sigt*wfactor
          sigtt=sigtt*wfactor
          siglt=siglt*wfactor
```

- **fitpar()** is an array of free parameters that will be determined as part of the L/T–iteration process.
- It is the functional dependence that needs to be determined first, not the parameters.
- However, it is also essential that you make some good guesses of initial parameter values for your first iteration.

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# Recalculating SIMC Weights

- You do not need to re–run SIMC every iteration if you have a way to over–write the weights
- Note, the weight is not $d^2\sigma/dtd\varphi$!
- It is proportional to $d^5\sigma/dE'd\Omega e'd\Omega\pi$ and corresponds to the expected rate for the input luminosity!

**See event.f**

```
! The total contributing weight from this event -- this weight is
! proportional to # experimental counts represented by the event.
! Apply survival probability to kaons if we're not modeling decay.

      main%weight = main%SF_weight*main%jacobian*main%gen_weight*main%sigcc
```

Event mapping Jacobian

$d^5\sigma$ from physics routine

```
!
! Note that there are also jacobians associated with some and/or all of
! the above.
! 1: We generate uniformly in xptar/yptar, not theta/phi.  We define the
! phase space volume (genvol contribution) as the product of the xptar/yptar
! range, and have a jacobian for each event taking into account the mapping
! between the solid angle on the unit sphere, and the dxptar/dyptar volume
! (the jacobian is 1/cos**3(dtheta), where dtheta is the angle between the
! event and the central spectrometer vector
```

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# Recalculating SIMC Weights

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- **You do not need to re–run SIMC every iteration if you have a way to over–write the weights**
- Need to retain all factors going from $d^2\sigma/dtd\varphi \rightarrow$ Weight when doing the re–calculation
- Hopefully you have access to one of Bill's Weight (ROOT/C++) re–calculation routines.
- This is my example from Fpi–2 LD2 analysis (PAW/Fortran):

```
lichen> more wt32.f
        function wtn(q2_set)
        include ?

        real p1,p2,p3,p4,p5,p6,p7,p8,p9,p10,p11,p12
        real nsigl,nsigt,nsiglt,nsigtt,tmp
        real nsig219,nsig,wtn
        real ft,tav,ftav

        real pi,mtar_gev,q2_gev
        real my_limit
        integer q2_set
        parameter (pi=3.14159)
        parameter (mtar_gev=0.93827231)

        if (abs(q2_set-245).lt.1) then
           p1=  0.28952E+02
           p2= -0.10000E+02
           p3= -0.15000E+02
           p4=  0.00000E+00
           p5=  0.46602E+02
           p6= -0.30000E+02
           p7=  0.18368E+01
           p8=  0.00000E+00
           p9=  0.10000E+04
           p10=-0.28000E+02
           p11= 0.35000E+01
           p12= 0.00000E+00
        else
           write(*,*)'wtn: q2 error ',q2_set
        endif
```

```
***
* Parameterization based upon Fpi-1 pi+ IT25, 12.04.18
* Revised for IT21, 12.11.06
*        tav=(0.0735+0.028*log(Q2i))*Q2i
        q2_gev=float(q2_set)/100.
        tav=(0.0735+0.028*log(q2_gev))*q2_gev
        ftav=(ti-tav)/tav
        ft=ti/(ti+0.139570**2)**2

*       nsigl=(p1+p2*log(Q2i))*exp((p3+p4*log(Q2i))*ti)
        nsigl=(p1+p2*log(Q2i))*exp((p3+p4*log(Q2i))*(ti-0.2))
        nsigt=p5+p6*log(Q2i)+(p7+p8*log(Q2i))*ftav

*       nsiglt=p9*exp(p10*ti)*sin(thetacmi)
        nsiglt=(p9*exp(p10*ti)+p11/ti)*sin(thetacmi)
        nsigtt=(p12*Q2i*exp(-Q2i))*ft*sin(thetacmi)**2

        nsig219=(nsigt+epsiloni*nsigl+epsiloni*cos(2.*phicmi)*nsigtt
     1      +sqrt(2.0*epsiloni*(1.+epsiloni))*cos(phicmi)*nsiglt)/1.d0

        wfactor=1.D0/(Wcmi**2-mtar_gev**2)**2
        nsig=nsig219*wfactor

        nsig=nsig/2./pi/1.d+06   !dsig/dtdphicm in microbarns/MeV**2/rad

        wtn=Weight*nsig/dsigdt

        my_limit=0.20
        if ((wtn.lt.my_limit).and.(wtn.gt.0.0)) then
          continue
        else
          wtn=0.
        endif

        return
        end
```

# It is crucial to keep organized

- **Each $Q^2$,W should be done separately.**
- **It is too much to expect the procedure to work globally, we only need to properly take into account the kinematic variation across a single diamond at a time.**
- Keep each iteration in a different directory, e.g. Q2_xx/IT_yy
- Keep ALL output. Don't throw anything away!
- Example fitpar() for $Q^2$=2.45 LD+ iteration #11

```
libra> cd it11/
libra> ls
par.pl_245
libra> cat par.pl_245
 0.82135E+03  0.12595E+03   1          4.1
-0.41000E+03  0.00000E+00   2          4.1
-0.24615E+02  0.12942E+01   3          4.1
 0.11100E+02  0.00000E+00   4          4.1
 0.35925E+02  0.18332E+01   5          0.4
-0.18000E+02  0.00000E+00   6          0.4
 0.27316E+02  0.62567E+01   7          0.4
-0.31000E+02  0.00000E+00   8          0.4
 0.00000E+00  0.00000E+00   9         47.4
-0.20000E+02  0.00000E+00  10         47.4
-0.12451E+03  0.98541E+01  11          1.2
 0.00000E+00  0.00000E+00  12          1.2
libra>
```

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# Evaluating Model Dependence

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- Blok et al.,
  PRC **78** (2008) 045202

- This is the Model Dependence to our L/T/LT/TT, not the Model Dependence to any final Form Factor values, which must be evaluated separately!
- **DO THIS AT THE END, AFTER YOUR CROSS SECTIONS ARE FINALIZED!**

Since the extracted separated cross sections depend in principle on the cross section model, there is a "model" systematic uncertainty. This uncertainty was studied by extracting $\sigma_L$ and $\sigma_T$ with different cross section models. Since the longitudinal and transverse cross sections in the model reproduce the experimental values to within 10%, these two terms were independently increased and decreased by 10% in the model. With these changes, the extracted $\sigma_L$ and $\sigma_T$ varied by less than 0.5%. For evaluating the model uncertainty due to the interference terms $\sigma_{LT}$ and $\sigma_{TT}$, these terms were independently increased or decreased by their respective uncertainties, obtained when fitting the four structure functions, and L/T separations were done with the modified models. The contribution to the uncertainty of $\sigma_L$ and $\sigma_T$ of these two terms is between 1% and 8% and depends strongly on $t$. The latter value (at the largest values of $-t$) is comparable to the contribution of uncorrelated uncertainties to $\sigma_L$ and $\sigma_T$.

# A few pointers

- Read over the Blok paper VERY CAREFULLY

- A single $Q^2$,W iteration should only take 1-2 hours
- I was able to do several iterations in a day, between teaching and other duties
- The fitpar() should converge in a few iterations to give 0.5<R<2
- The main work is in getting R to be acceptably flat and in getting stable cross sections

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