NPS-Elastic Calibrations

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CONTENTS

I. INTRODUCTION

The Neutral Particle Spectrometer (NPS) calorimeter consists of 30 columns and 36 rows of (2 cm)^2 $\mathrm{PbWO_4}$ crystals. The energy and spatial resolution of the calorimeter will be calibrated by elastic $\mathrm{H}(e, e'_{\mathrm{Calo}} p_{\mathrm{HMS}})$ events. In addition, the NPS DVCS program requires running the Hall C High Momentum Spectrometer (HMS) at central momenta > 5 GeV/c. The optics of the HMS at these high momentum settings requires recalibration. Measuring the momentum scale of the HMS focal plane requires taking single arm $H(e, e'_{\mathrm{HMS}})p$ elastic data at the required momenta.

II. CALORIMETER CALIBRATION

The $\mathrm{PbWO_4}$ calibration requires finding appropriate $\mathrm{H}(e, e'_{\mathrm{Calo}} p_{\mathrm{HMS}})$ kinematics such that the coincidence count rate is reasonably high, the acceptances of the calorimeter and HMS focal plane are balanced, and the configuration is mechanically achievable. A critical challenge is to achieve full vertical illumination of the calorimeter.

A. Vertical Acceptance Matching

I use a coordinate system with $\hat{z}$ along the beam direction, $\hat{y}$ vertical, and $\hat{x} = \hat{y} \times \hat{z}$. For a proton of momentum P detected in the HMS with horizontal and vertical angles θ_H and θ_V, respectively, relative to HMS central ray, the momentum components are

$$\mathbf{P} = P\left[-\sin(\Theta_{\text{HMS}} + \theta_H), \sin(\theta_V), \sqrt{1 - \sin^2(\Theta_{\text{HMS}} + \theta_H) + \sin^2(\theta_V)}\right] \tag{1}$$

with $\Theta_{\text{HMS}} > 0$ the central angle of the spectrometer (located in the $-x$ side of the beam), and θ_H is chosen positive when it points away from the beam line. The azimuthal angle ϕ_P of the proton around the beamline line is

$$\begin{aligned} \phi_P &= \text{atan2}(\sin(\theta_V), -\sin(\Theta_{\text{HMS}} + \theta_H)) \\ \tan\phi_P &= -\frac{\sin(\theta_V)}{\sin(\Theta_{\text{HMS}} + \theta_H)} \end{aligned} \tag{2}$$

To simplify, consider the case $\theta_H = 0$:

$$\begin{aligned} \tan\phi_P &\to \frac{\sin(\theta_V)}{-\sin(\Theta_{\text{HMS}})} \\ \phi_P &\approx \pi - \frac{\theta_V}{\sin(\Theta_{\text{HMS}})} \end{aligned} \tag{3}$$

In a coincidence $\text{H}(e, e'p)$ event, the beam, scattered electron, and recoil proton momenta are $\mathbf{k}$, $\mathbf{k}'$ and $\mathbf{P}$, respectively. Momentum conservation requires $\mathbf{k} - \mathbf{k}' = \mathbf{P}$. In particular, the out-of-plane momentum components balance:

$$\mathbf{k}' \cdot \hat{y} = \mathbf{P} \cdot \hat{y} \tag{4}$$

Since the three vectors lie in a common plane, the azimuthal angles of $\mathbf{k}'$ and $\mathbf{P}$ balance:

$$\phi_{k'} = \pi + \phi_P \tag{5}$$

Both eqs. 4 and 5 demonstrate that the vertical angles $|\theta_V(P)|$ and $|\theta_V(k')|$ do not match, but are instead related approximately by

$$\frac{\theta_V(k')}{\sin(\Theta_e)} \approx \frac{-\theta_V(P)}{\sin(\Theta_{\text{HMS}})} \tag{6}$$

Given the large vertical acceptance of the NPS calorimeter, it is easiest to balance the acceptance by minimizing Θ_{HMS}. However in elastic scattering, smaller values of the proton recoil angle are only achieved by increasing the Q^2, and thereby reducing the cross section.

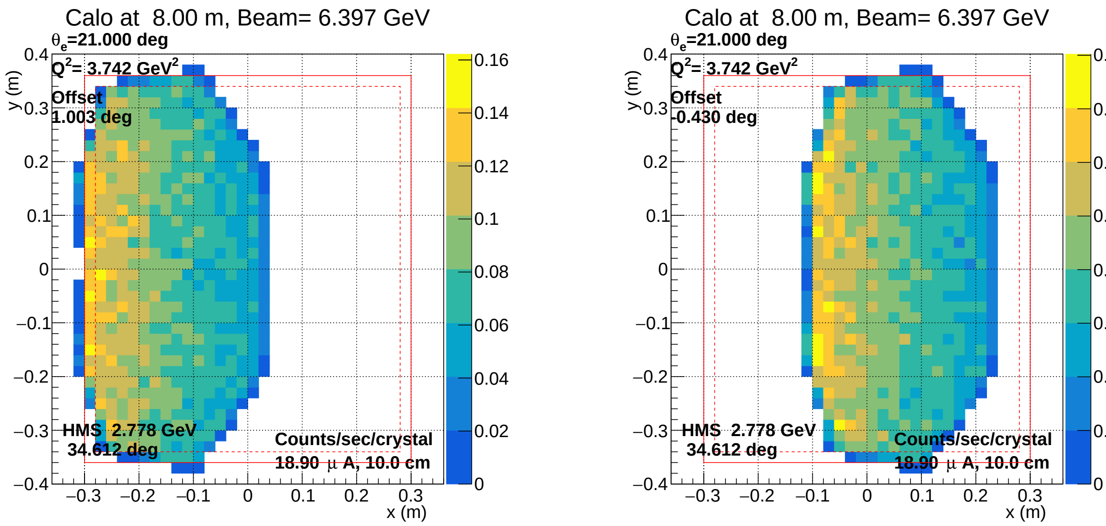
B. Three-Pass Elastic Calibration Kinematics

TABLE I. Coincidence elastic ep kinematics at 3-pass beam $k = 6.397$ GeV.

Proton Momentum, $\lvert\mathbf{P}\rvert$, in HMS	2.778 GeV
HMS Central Angle	34.612°
Central Calorimeter (SHMS) Angle	21.00°
Nominal Electron Momentum	4.403 GeV
Nominal Q^2	3.742 GeV^2
Three SHMS angle settings	21.003°, 20.570°, 19.138°
Count Rate per setting @ 18.9 μA	44/sec
Total calibration time @ 30 μA	7.5 hour

For the 2023 NPS run, the expected three-pass energy in Hall C is 6.397 GeV. With the calorimter face (or perhaps shower max?) at 8.00 m from the pivot, a nominal kinematic setting for coincidence elastic calibrations is shown in Table I. The full coverage of the calorimeter is achieved with the three shifted calorimeter settings illustrated in Fig. 1. With 2.5 hours of beam at 30 μA at each setting, every crystal in fiducial volume of the calorimeter (dashed box) will register at least 1000 events with the centroid of the EM shower in the crystal.

A possible configuration for 5-pass calibration is shown in Fig. 2. Again, three slightly shifted settings of the SHMS carriage will completely illuminate the calorimeter. In this case 2.5 hours per setting at 30 μA will achieve ≥ 280 counts per block. For each crystal, there will be additionally $>\sim 1100$ events with the centroid of the shower in one of the 8 adjacent crystals.



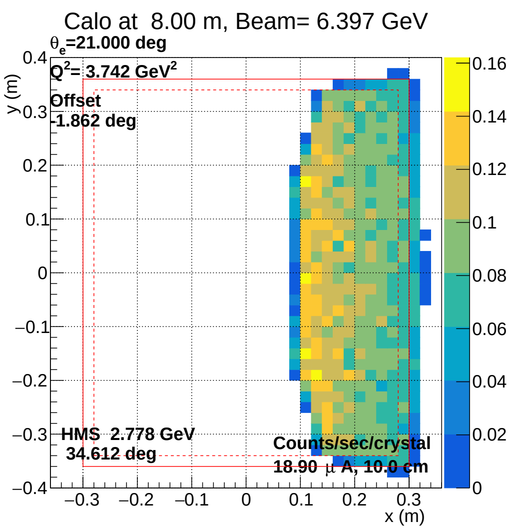


FIG. 1. Three-pass *ep* coincidence kinematics. Each plot shows the footprint of scattered electrons in the NPS calorimeter, in coincidence with fixed HMS proton kinematics. The central ray of the HMS is set to $|\mathbf{P}| = 2.778$ GeV at 34.612 degrees. The central angle of the SHMS carriage for each setting is 21.0° plus the Offset value on the plot. The color scale is the $(e, e'p)$ coincidence elastic count rate per second per $PbWO_4$ crystal, with 20 μA incident on a 10 cm LH2 target.

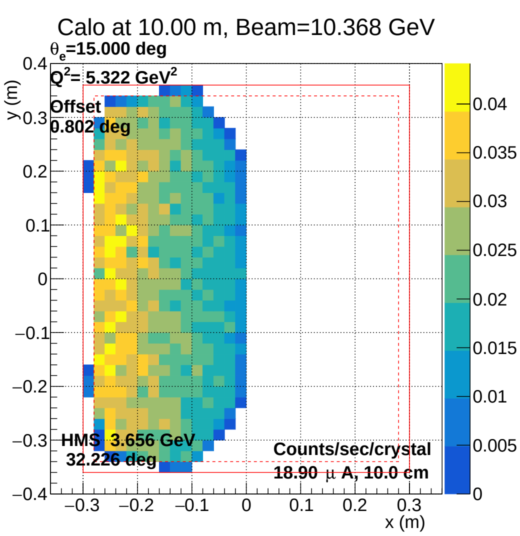


FIG. 2. Five-pass *ep* coincidence kinematics. The plot shows the footprint of scattered electrons in the NPS calorimeter (located at 10 m), in coincidence with HMS proton detection. The color scale is the $(e, e'p)$ coincidence elastic count rate per second per $PbWO_4$ crystal, with 20 μA incident on a 10 cm LH2 target.

TABLE II. HMS Elastic ep Calibration Settings. Count rate is at a luminisity of $5 \cdot 10^{37}/\mathrm{cm}^2/\mathrm{sec}$ (Beam current 19.8 μAmp).

Setting	1	2	3	4	5
Beam (GeV)	10.558	10.558	10.558	10.558	10.558
k'_{HMS} (GeV)	6.934	6.117	6.028	5.442	5.229
θ_{HMS} (deg)	17.531	20.694	21.058	23.587	24.573
Events (1000/hr)	31.5	6.66	5.67	2.06	1.44

III. HMS MOMENTUM CALIBRATION

There are five HMS settings above 5 GeV in the NPS run plan. Calibrating the HMS optics requires DIS runs with the sieve slit. The HMS momentum scale will be calibrated with elastic cross section measurements. The five settings requiring HMS calibration are listed in Table II, together with angular settings for elastic calibrations. The table also includes the elastic $\mathrm{H}(e,e')p$ count rate at $\sim 20\mu$Amp beam current on a 10 cm LH2 target. Fig. 3 displays the distribution of elastic events in the HMS δ *vs.* θ_H plane for settings 4 and 5 of Table II. Additional elastic data can be taken with HMS angle offset by ± 25 mrad (same momentum setting) to calibrate correlations between δ and θ_H. Entire time to complete all five elastic settings is ~ 1 shift.

Appendix A: Elastic ep Cross Section

The elastic ep cross section has the form:

$$
\begin{aligned}
\frac{d\sigma}{d\Omega_e} &= \sigma_{\mathrm{Mott}} \frac{1}{[1+\tau]} \left[\frac{\tau}{\epsilon} G_M^2(Q^2) + G_E^2(Q^2)\right] \\
\sigma_{\mathrm{Mott}} &= \left[\frac{\alpha_{\mathrm{QED}} \cos(\theta_e/2)}{2k \sin^2(\theta_e/2)}\right]^2 \frac{k'}{k} \\
\epsilon &= \left[1 + 2(1+\tau)\tan^2(\theta_e/2)\right]^{-1}, \qquad \tau = \frac{Q^2}{4M^2}
\end{aligned}
\tag{A1}
$$

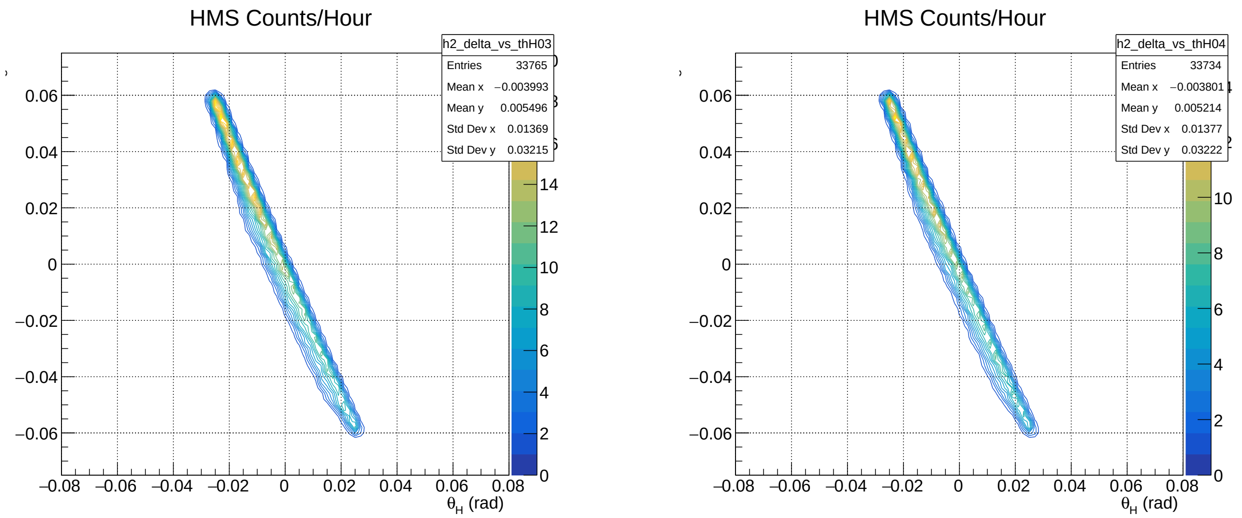


FIG. 3. Five-pass HMS elastic calibration kinematics (Table II. The color scale is the H(e, e')p elastic count rate per hour, with 20 μA incident on a 10 cm LH2 target. **Left:** HMS central momentum 5.442 GeV. **Right:** HMS central momentum 5.229 GeV.

For the Form Factors, I used the parameterization of J.Kelly, Phys Rev C **70** (2004) 068202:

$$G_E(Q^2) = \frac{1 + a_1^E\,\tau}{1 + \sum_{n=1}^{3} b_n^E\,\tau^n}$$

$$G_M(Q^2) = \mu_p \frac{1 + a_1^M\,\tau}{1 + \sum_{n=1}^{3} b_n^M\,\tau^n} \tag{A2}$$

The fitted parameter are listed in Table III.

Eq. A1 does not explicitly include the effects of two-photon exchange. However, these effects are partially subsumed into the parameters of Table III, as they were fitted to the cross section data without removing two-photon exchange effects.

TABLE III. Proton Elastic Form Factor Parameters

	a_1	b_1	b_2	b_3
G_E	-0.24 ± 0.12	10.98 ± 0.19	12.82 ± 1.1	21.97 ± 6.8
G_M	0.12 ± 0.04	10.97 ± 0.11	18.86 ± 0.28	6.55 ± 1.2