KaonLT Meeting

June 26nd, 2025

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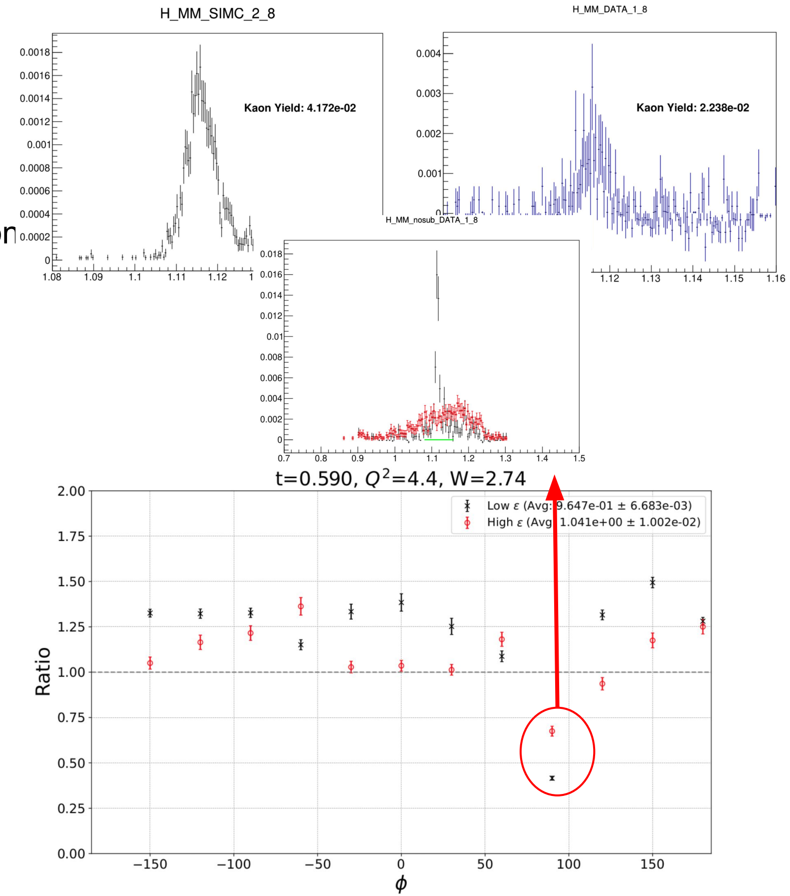
Recent Fixes/Improvements since mass correction in theta

Pion Subtraction Uncertainty – Bug Fix

- Discovered a bug affecting the calculated values for pion subtraction uncertainty
- Fix has been implemented
    - Uncertainty is now properly calculated and propagated
- The pion subtraction was also improved a bit. There was oversubtraction in some bins which explain some of the “bad ϕ bins” in the ratio plots
    - Note: there are still some dips in ϕ bins
    - For instance, t=0.590, Q2=4.4, W=2.74, ϕbin=9 has a dip, but if you look at the right plots you can see there is a bit of over subtraction (pion in red) that needs to be adjusted.

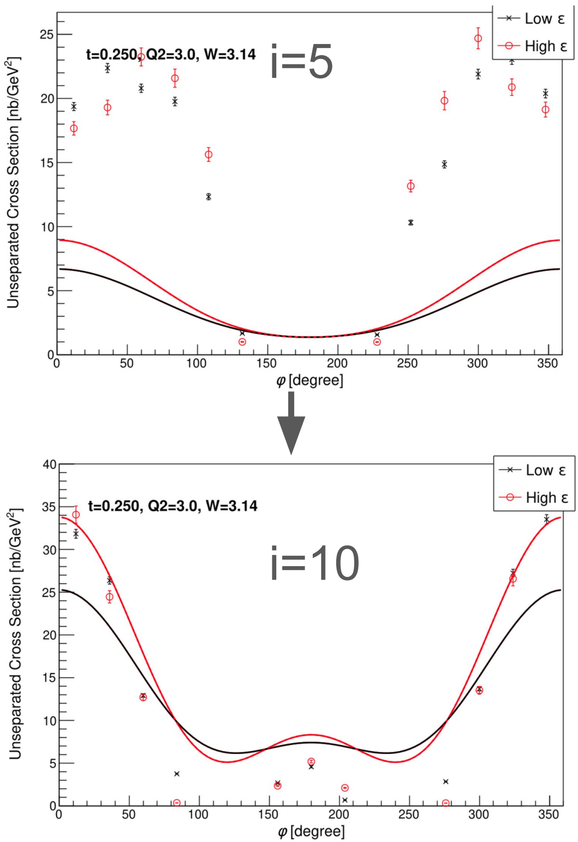
Center Low



Recent Fixes/Improvements since mass correction in theta

LT Separation Fit Stability – Physics-Constrained Improvement

- This is a significant methodological improvement
  - Important implications for fit independence and physical diagnostics
- Issue: The LT separation code (i.e., extraction of L, T, LT, TT from a simultaneous fit):
  - Was hitting fit bounds, especially during iterative fitting
  - This introduced biases and drove separated cross sections (L, T) to unphysical values
- Initial fix:
  - Imposed simple physical boundary conditions: L, T > 0
- Further improvement:
  - Derived a better-constrained fit using the Cauchy-Schwarz Inequality
    - Bakes the positivity constraint into the fit itself, so there's never a step with σ_L < 0 or σ_T < 0
- By iterating, the model is now pushed out of unphysical local minima
- For instance…
  - t-bin = 0.25, Q^2 = 3.0, W = 3.14 → see right plots
    - Iteration 5 to 10 greatly improves data and fit agreement
  - This is expected as the fit is constrained to physical bounds now and thus the model must iterate away from an unphysical regime



Derivation of LT unseparated fit

- Dieter Drechsel, Lothar Tiator; J. Phys. G: 18 (1992)
- Using these relations, for the response functions (i.e., L/T/LT/TT unpolarized)
- Eqn. 20-23, the W terms are **Hadronic Tensors** so using the Cauchy-Schwarz inequality we can derive a new formulation for the response functions (i.e., separated cross sections).

$$\frac{d\sigma}{d\Omega_f\, d\varepsilon_f\, d\Omega_\pi} = \Gamma \frac{d\sigma_v}{d\Omega_\pi} \tag{19}$$

$$\frac{d\sigma_v}{d\Omega_\pi} = \frac{d\sigma_T}{d\Omega_\pi} + \varepsilon_L \frac{d\sigma_L}{d\Omega_\pi} + [2\varepsilon_L(1+\varepsilon)]^{1/2} \frac{d\sigma_{TL}}{d\Omega_\pi}\cos\Phi_\pi + \varepsilon \frac{d\sigma_{TT}}{d\Omega_\pi}\cos 2\Phi_\pi + h[2\varepsilon_L(1-\varepsilon)]^{1/2}\frac{d\sigma_{TL'}}{d\Omega_\pi}\sin\Phi_\pi + h(1-\varepsilon^2)^{1/2}\frac{d\sigma_{TT'}}{d\Omega_\pi} \tag{20}$$

where

$$\Gamma = \frac{\alpha}{2\pi^2}\frac{\varepsilon_f}{\varepsilon_i}\frac{k_\gamma}{Q^2}\frac{1}{1-\varepsilon} \tag{21}$$

is the flux of the virtual photon field. In this expression we have introduced the 'photon equivalent energy', $k_\gamma = (W^2 - m_i^2)/2m_i$, the laboratory energy necessary for a real photon to excite a hadronic system with CM energy W. The first two terms within parentheses on the RHS of equation (20) are referred to as the transverse (T) and longitudinal (L) structure functions. They do not depend on the azimuthal angle and may be decomposed into a multipole series in $\cos\Theta_\pi$. The third term and the fifth term describe transverse–longitudinal interferences (TL and TL′), due to their dependence on $\cos\Phi_\pi$ and $\sin\Phi_\pi$ they have to contain an explicit factor $\sin\Theta_\pi$, i.e. they vanish along the axis of momentum transfer. The same is true for the fourth term, a transverse–transverse interference (TT) proportional to $\sin^2\Theta_\pi$. The last term (TT′) can only be observed by target or recoil polarization (see section 2.6).

Particularly in the case of multipole decompositions it is useful to express the angular distribution of the emitted particle in the hadronic CM frame of the final state. Therefore, equation (19) should be interpreted with the flux factor in laboratory coordinates, while the virtual photon cross sections have to be evaluated in the CM frame. For the rest of this section we will only use CM variables. The transformation of the differential cross section to the laboratory frame is given in appendix A.

The virtual photon cross sections may be expressed in terms of the hadronic tensors W_{ik} by

$$\frac{d\sigma_v}{d\Omega_\pi} = \frac{|\boldsymbol{k}|}{k_\gamma^{CM}}\left(\frac{W_{xx}+W_{yy}}{2} + \varepsilon_L W_{zz} - [2\varepsilon_L(1+\varepsilon)]^{1/2}\,\mathrm{Re}\,W_{xz} + \varepsilon\frac{W_{xx}-W_{yy}}{2} + h[(2\varepsilon_L(1-\varepsilon)]^{1/2}\,\mathrm{Im}\,W_{yz} + h(1-\varepsilon^2)^{1/2}\,\mathrm{Im}\,W_{xy}\right) \tag{22}$$

where $k_\gamma^{CM} = (m_i/W)k_\gamma$ is the 'photon equivalent energy' in the CM frame.

A comparison of equations (20) and (22) suggests to introduce the response functions

$$R_T = \tfrac{1}{2}(W_{xx}+W_{yy}) \qquad R_L = W_{zz}$$
$$\cos\Phi_\pi R_{TL} = -\mathrm{Re}\,W_{xz} \qquad \sin\Phi_\pi R_{TL'} = \mathrm{Im}\,W_{yz} \tag{23}$$
$$\cos 2\Phi_\pi R_{TT} = \tfrac{1}{2}(W_{xx}-W_{yy}) \qquad R_{TT'} = \mathrm{Im}\,W_{xy}.$$

Derivation of LT unseparated fit

- This new formulation is only true when the boundary condition requires L,T > 0.
- Thus, using this Cauchy-Schwarz formulation we restrict the iterations to only physically real fits
  - We avoid boundary issues in the fits (e.g., LT or TT are artificially inflated because the fits are stuck at the boundaries) and unphysical extrema
  - Assures that L and T drive the fit, not LT and TT
- Discrepancies in data versus fit are reflected in the extracted separated cross section uncertainties

$$\rho_{LT} \equiv \frac{\sigma_{LT}}{\sqrt{\sigma_T \sigma_L}} \leq 1.0$$

$$\rho_{TT} \equiv \frac{\sigma_{TT}}{\sigma_T} \leq 1.0$$

$$\sigma = \sigma_T + \epsilon\sigma_L + \sqrt{2\epsilon(1+\epsilon)} \cdot \rho_{LT} \cdot \sqrt{\sigma_L\sigma_T}\cos\Phi + \epsilon \cdot \rho_{TT} \cdot \sigma_T \cos^2\Phi$$

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Past model

$$\sigma_L = (p_1 \cdot f_t) \cdot e^{-p_2|t|} \quad \omega(W)$$

$$\sigma_T = \frac{p_5}{|t|^{p_6}} \quad \omega(W)$$

$$\sigma_{LT} = p_9 \cdot e^{-p_{10}|t|} \sin\theta \quad \omega(W)$$

$$\sigma_{TT} = \frac{p_{13}}{|t|^{p_{14}}} \sin^2\theta \quad \omega(W)$$

$$\omega(W) = \frac{1}{(W^2 - M^2)^{0.85 \cdot W_c^2 - 5.97 \cdot W_c + 12.68}}$$

Q2=4.4, W=2.74 Cauchy-Schwarz fit results

New functional forms

Constants:

$$\pi, \quad m_{\text{tar}} = 0.93827231, \quad m_{\pi+} = 0.139570, \quad m_{K+} = 0.493677$$

$$t_{\text{av}} = \left(0.05032 + 0.01345 \ln Q^2_{\text{set}}\right) Q^2_{\text{set}},$$

$$f_{\text{tav}} = \frac{|t| - t_{\text{av}}}{t_{\text{av}}},$$

$$f_t = \frac{|t|}{\left(|t| + m^2_{K+}\right)^2},$$

$$\sigma_L = (p_1 f_t) \exp(-|p_2 t|),$$

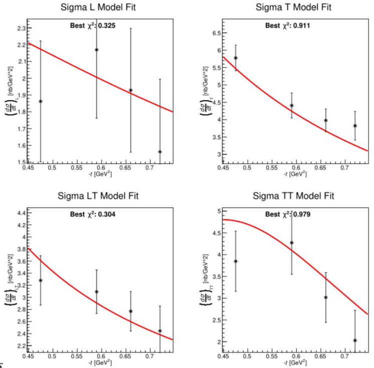
$$\sigma_T = \frac{p_5}{|t|^{p_6}} \exp(-|p_7 t|),$$

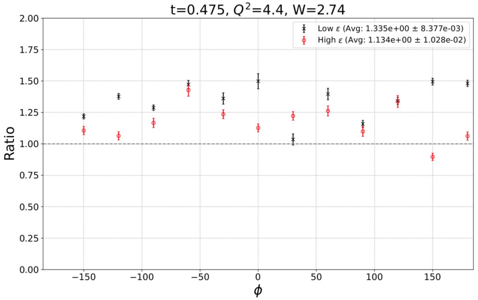
$$\sigma_{LT} = \frac{p_9}{|t|},$$

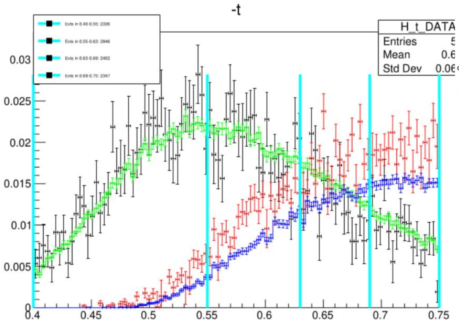
$$\sigma_{TT} = \frac{p_{13}}{|t|^{p_{14}}} \exp(-|p_{15} t|),$$

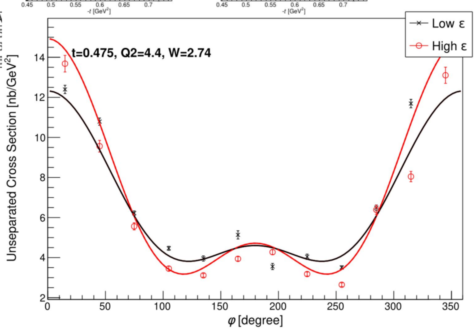
→ No sine terms

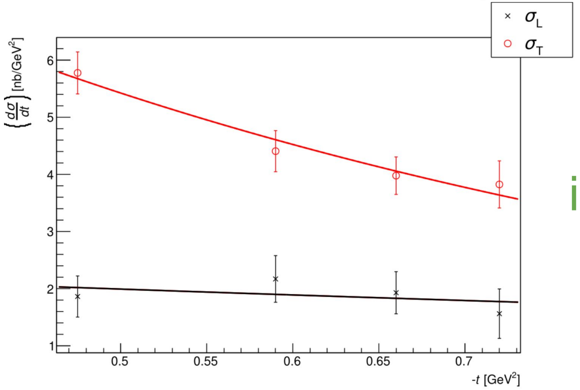
$$w_{\text{factor}} = \frac{1}{\left(W^2 - m^2_{\text{tar}}\right)^{0.85 W^2_{\text{set}} - 5.97 W_{\text{set}} + 12.68}}.$$





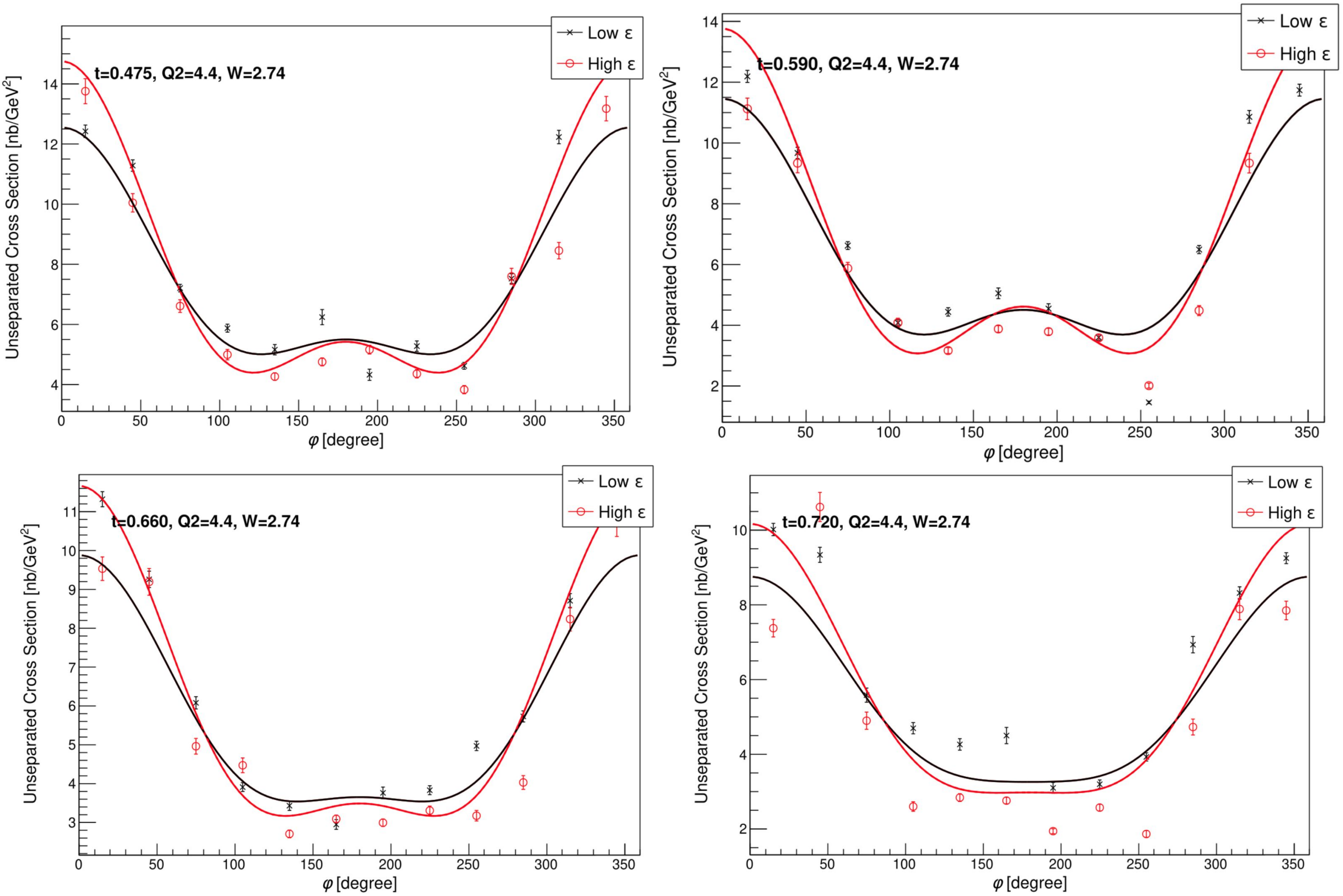






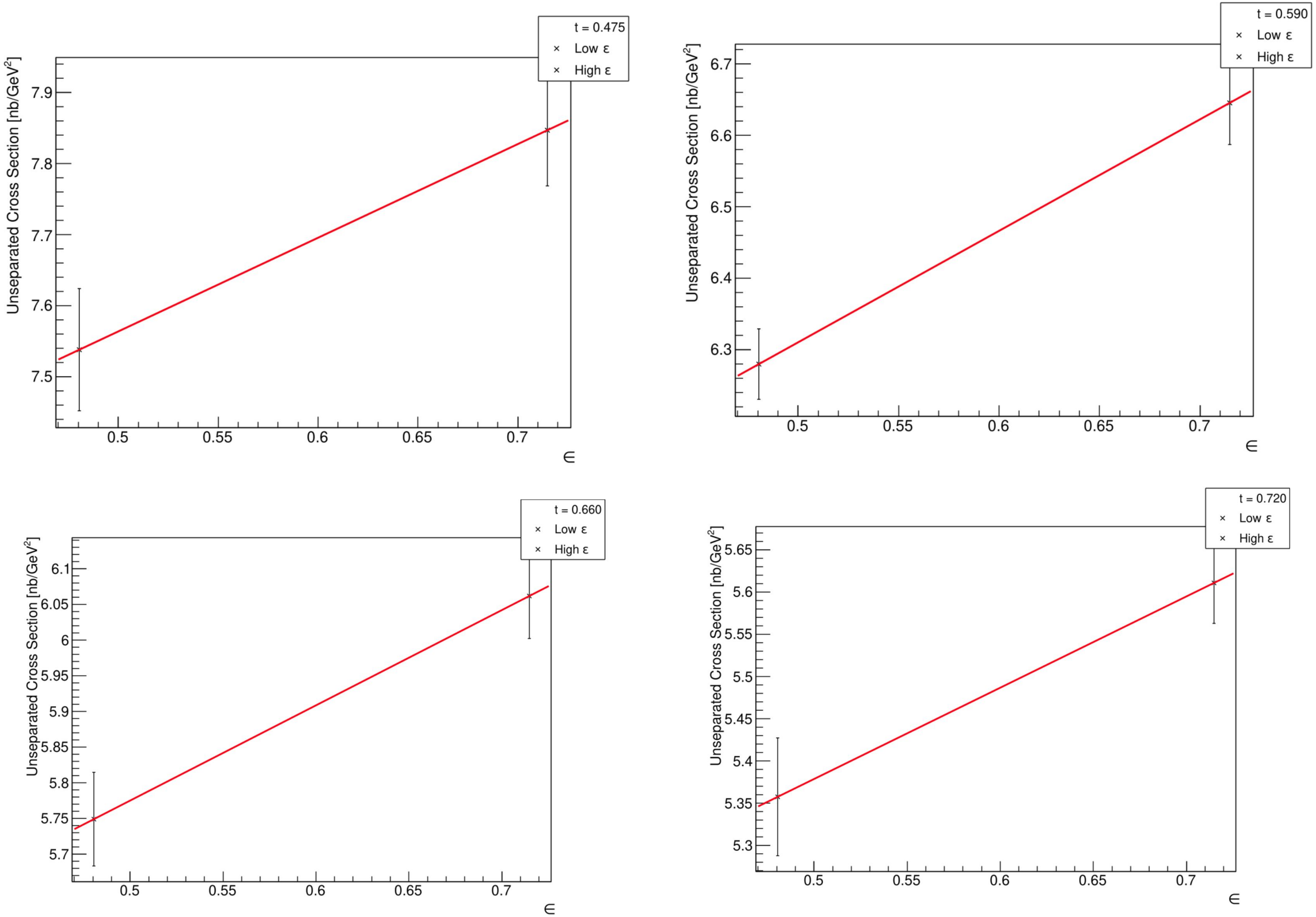
i=6

Q2=4.4, W=2.74 Cauchy-Schwarz fit results



20 iterations

Q2=4.4, W=2.74 Cauchy-Schwarz fit results



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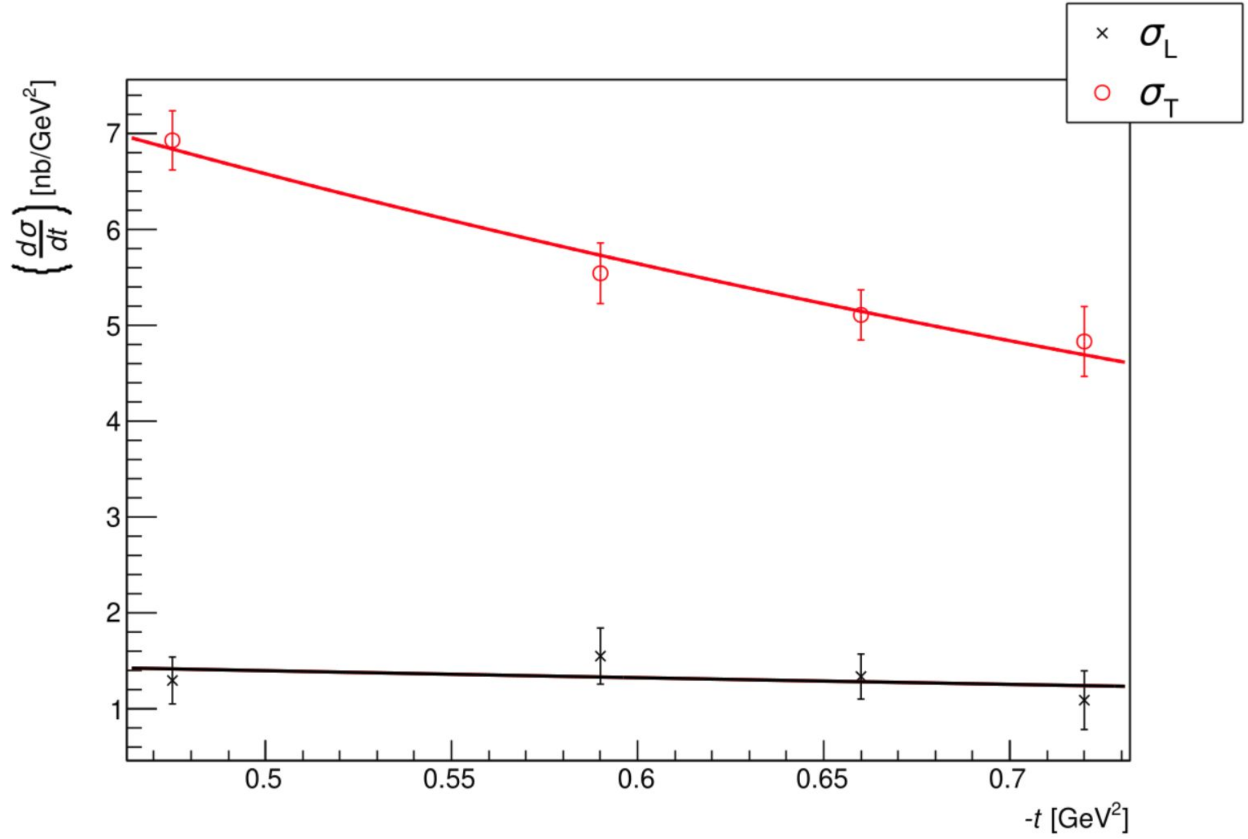

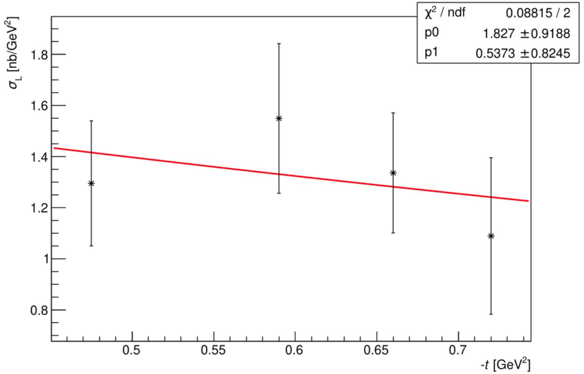

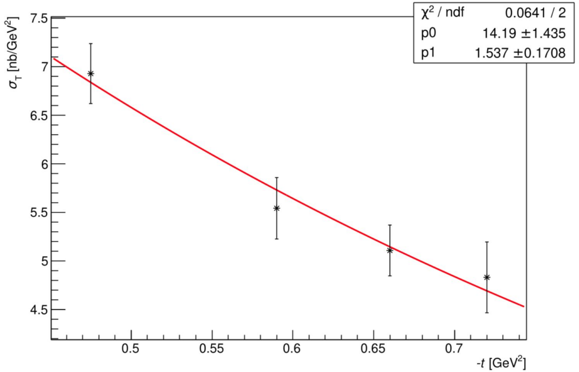

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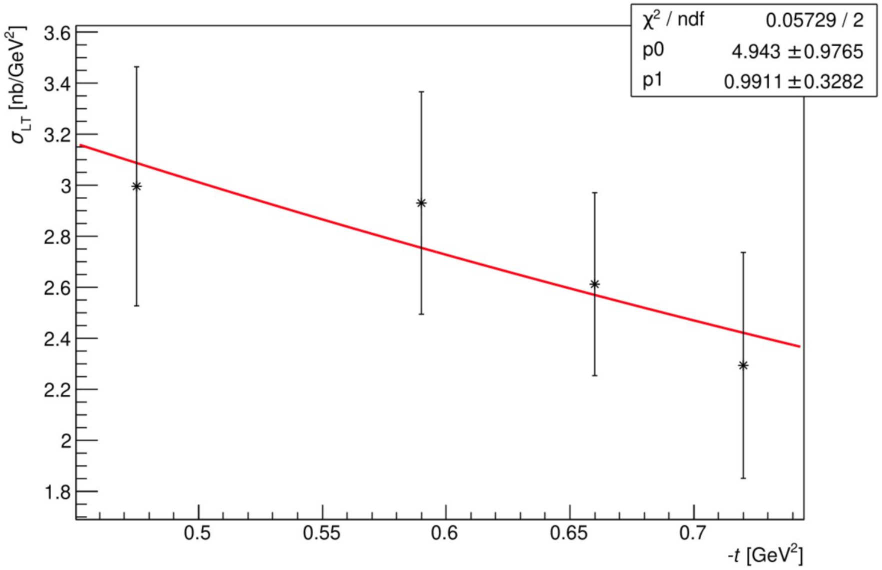

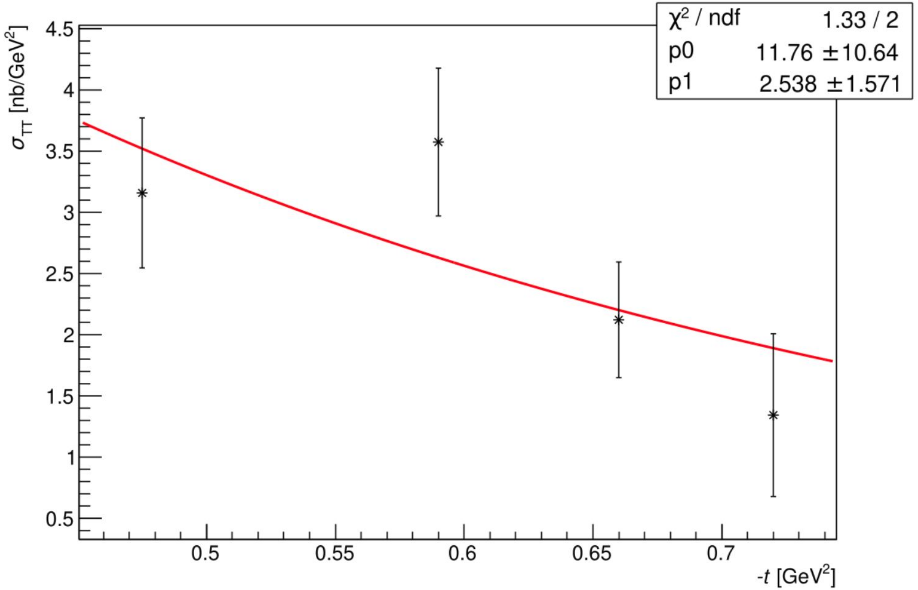

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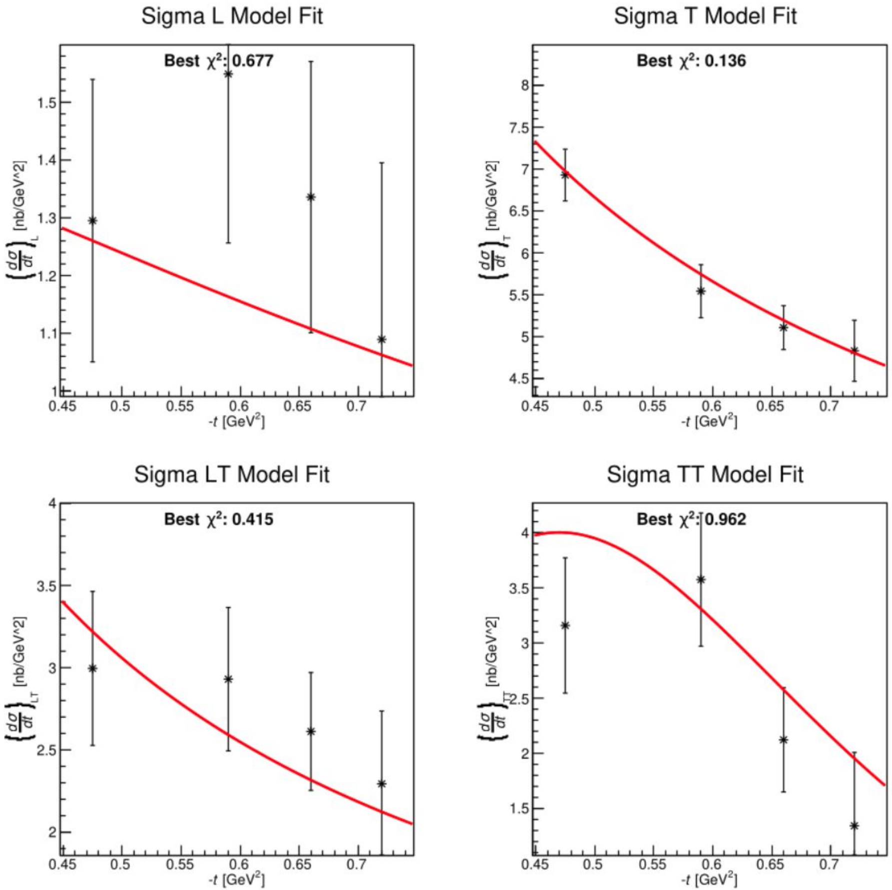

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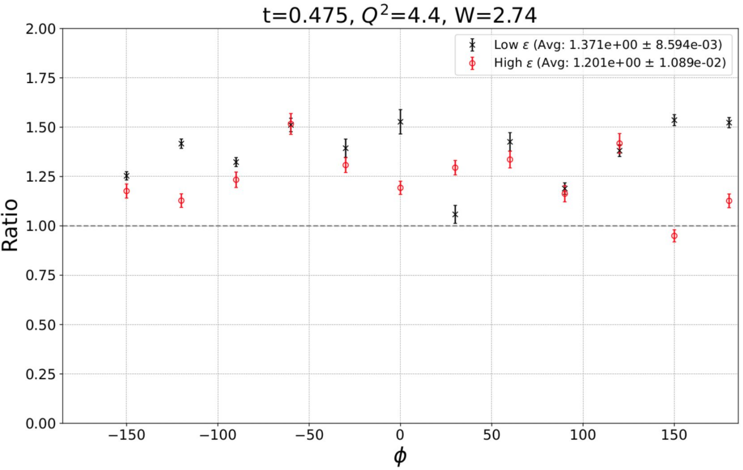

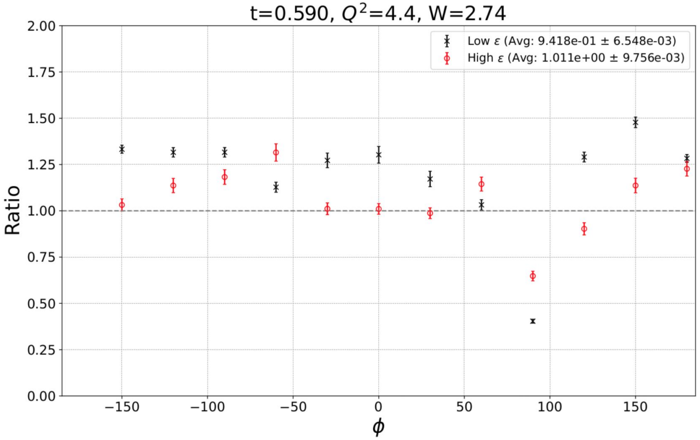

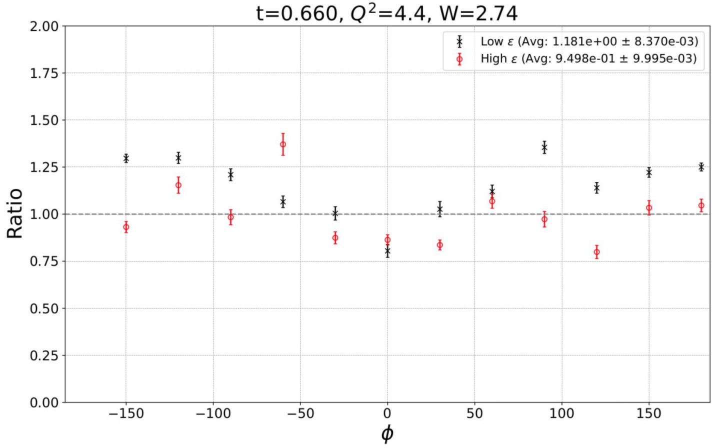

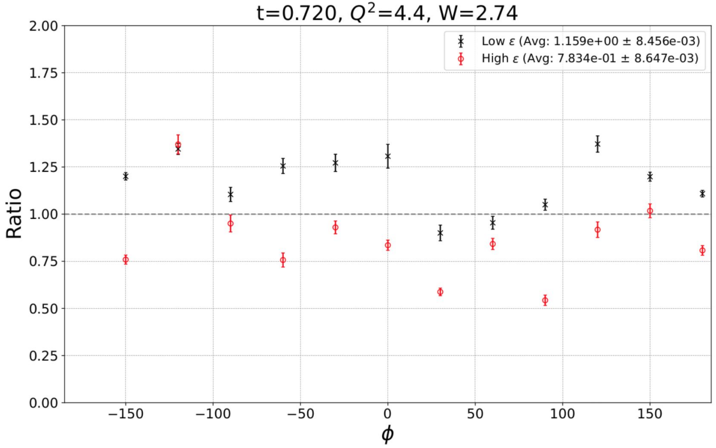

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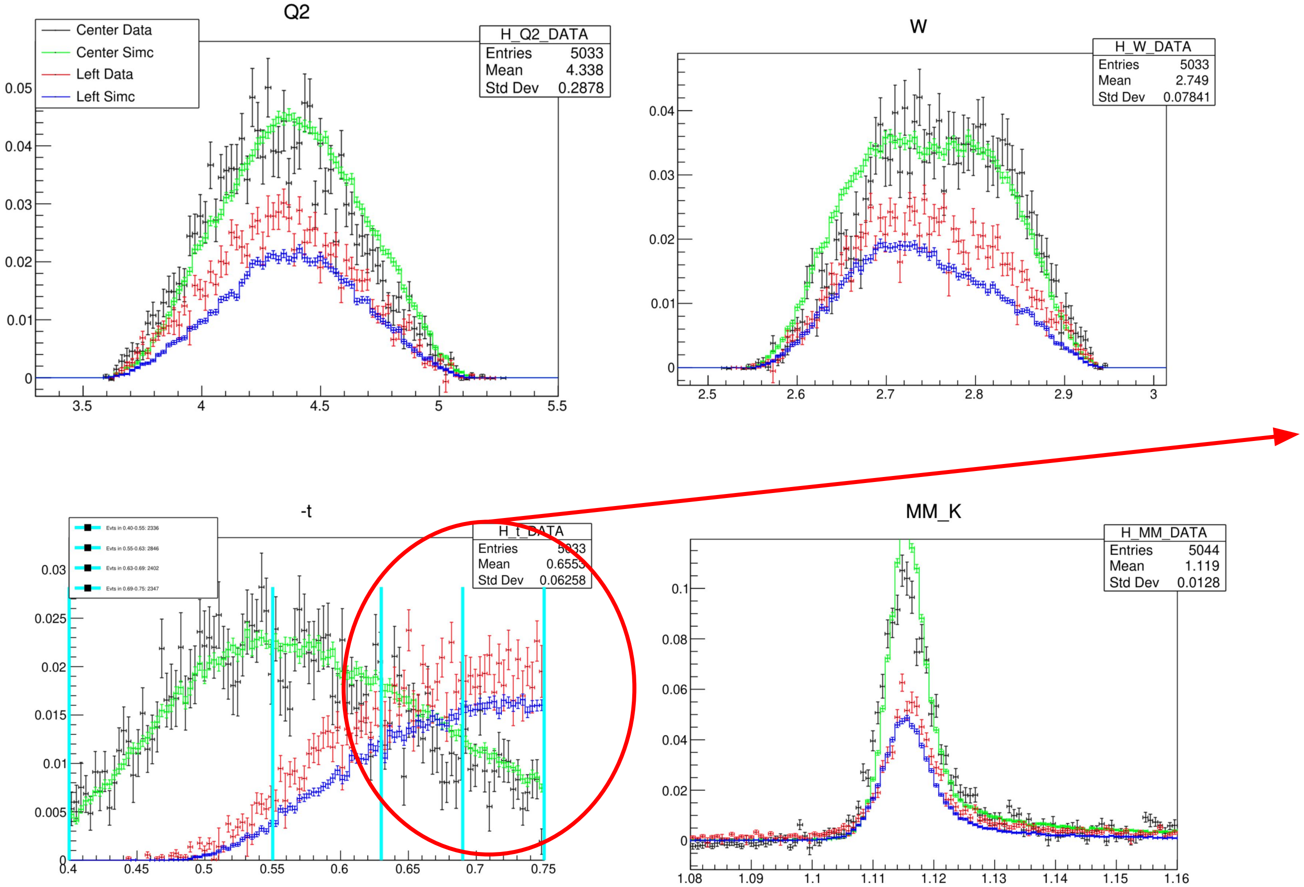
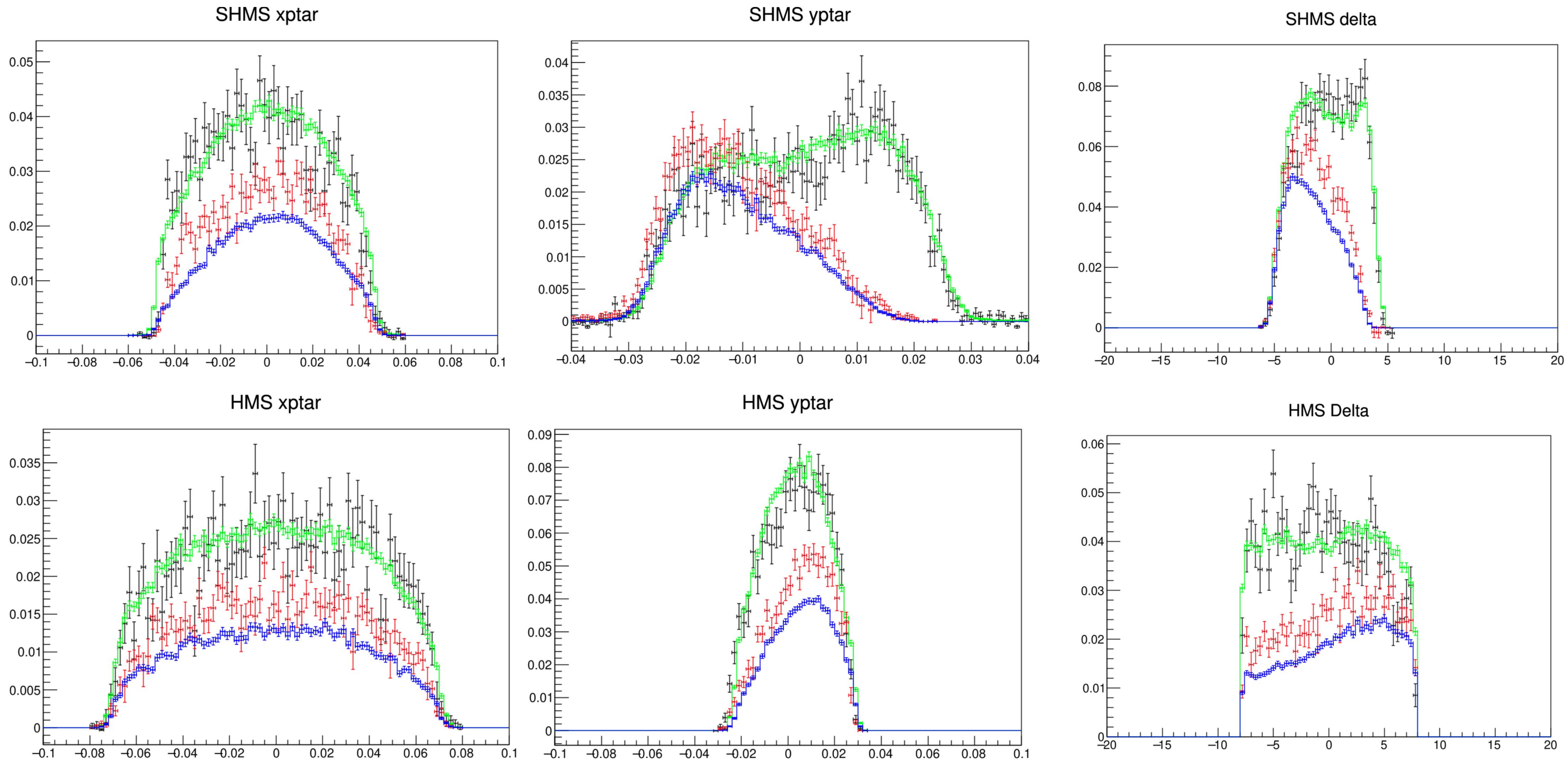


Most left setting events are at high |-t|

Q2=4.4, W=2.74 Cauchy-Schwarz fit results



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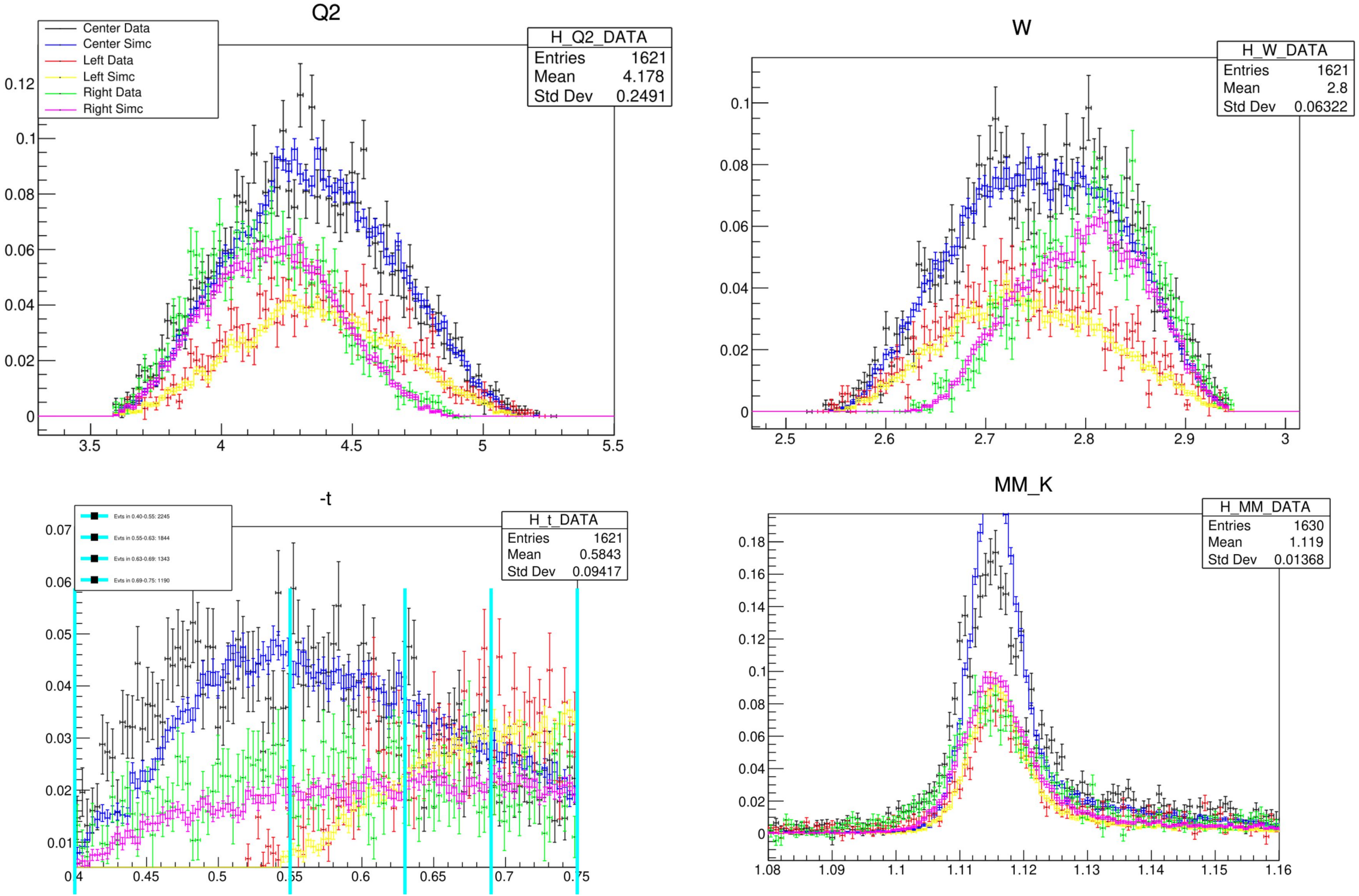

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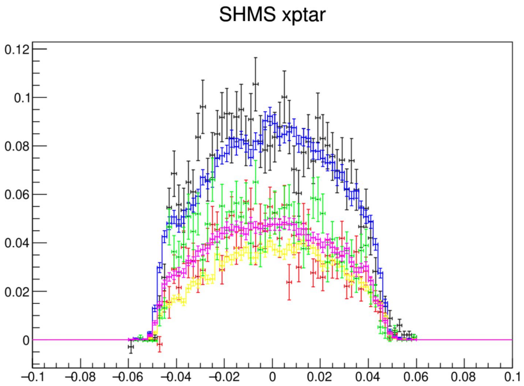
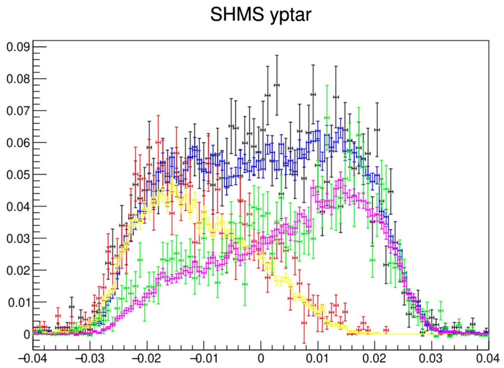
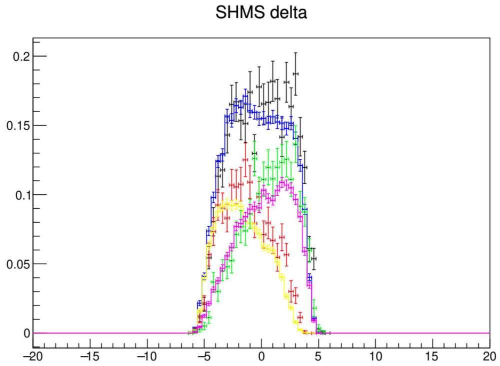
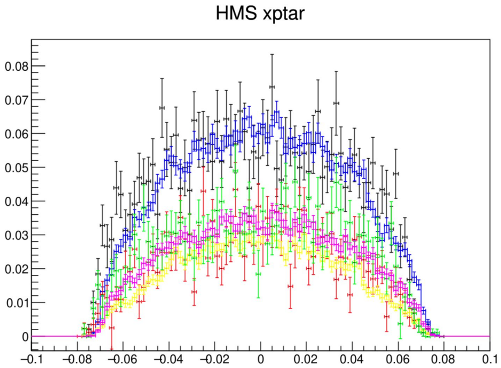
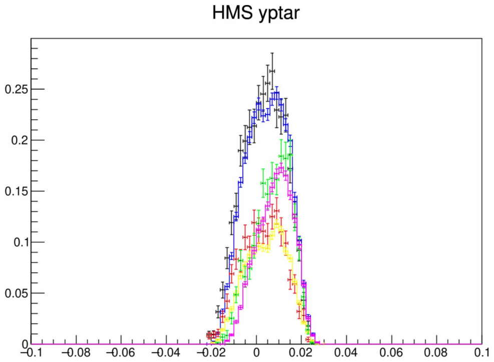
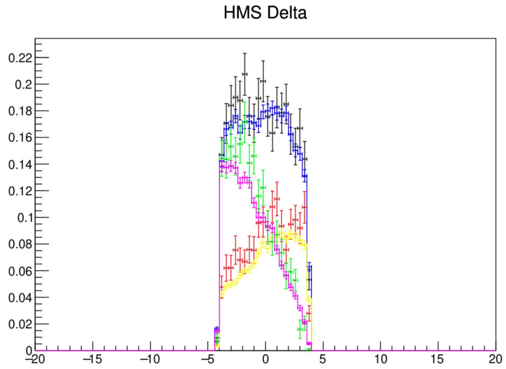












Adjustments per Setting

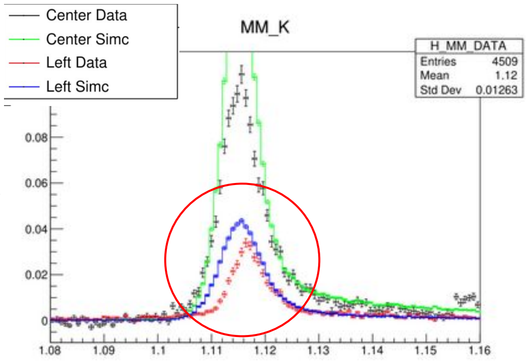
Overall, there is a similar set of steps for each setting. I'll highlight some of the areas that need specific investigation

- Q2=3.0, W=2.32
  - Adjust pion background subtraction scaling for bad t-ϕ bins (4-5 bad bins)
  - Further tweaks to model
- Q2=3.0,W=3.14
  - Investigate Left setting MM (this is true for BOTH high and low epsilon)
  - Adjust pion background subtraction scaling for bad t-ϕ bins (2-3 bad bins)
  - Further tweaks to model
- Q2=4.4, W=2.74
  - Adjust pion background subtraction scaling for bad t-ϕ bins (1 bad bin)
- Q2=5.5, W=3.02
  - Further tweaks to model

After all settings have a good set of fits, rerun SIMC and refine model setting by setting

Final Statements (1)

- The main issues across all settings are…
  - Adjust pion background subtraction scaling for bad t-ϕ bins
    - It is very clear in the ratio plots where these are and small adjustments to the pion scaling factor for these bins bring outlines in line with the rest of data
  - Adjust model
    - LT/TT
    - Theta terms
- Q2=3.0, W=3.14 still has this weird issue with MM of data vs SIMC
  - This is only the case for Left, both high and low epsilon. That means two different run periods (and energies) which adds to the mystery.
  - I tried fixing the MM shift to center it at lambda mass, but the width of the MM peaks are oddly skinny compared to all other settings.
  - Current theory is a typo or wrong value in SIMC, needs further investigation



Final Statements (2)

- Statistical uncertainties
  - The statistical uncertainties per setting vary a bit, but generally fall in the 1.5-2.5% range. Now, this will improve when I rerun the final parameterization with many SIMC events so I expect this to fall to around 0.5-1.5%
- Systematic uncertainties
  - As I said in my previous email, I have a good handle on the point-to-point systematics, primarily driven by the HGCer PID. My estimate for pt-to-pt is currently ~3–3.5%. For all settings, I assume a fixed 3.6% for these plots
  - The t-correlated contributions are expected to exceed the PAC’s 2.0% at high t-bins, while lower t-bins are likely close to that value, possibly slightly higher.
  - The global scale uncertainty remains to be finalized.

EXTRA

Studies of L, T > 0

- Wij is a positive-semidefinite submatrix of the hadronic tensor.
  - Any violation indicates a breakdown of fundamental assumptions (unitarity, hermiticity, one-photon exchange).
- Positivity implies a valid one-photon–exchange framework; observing $\sigma_T<0$ or $\sigma_L<0$ would signal an inconsistency
- The hadronic tensor's positive-semidefiniteness (and hence $|\rho_{LT',TT}|\leq 1$) follows directly from the assumption of a single, real virtual-photon probe
-