1 Dieter Drechsel, Lothar Tiator; J. Phys. G: 18 (1992)

The Hadronic tensor

$$W_{\mu,\nu} = (\frac{M}{4\pi\omega})^2 < \chi_f|J_\mu|\chi_i > \cdot < \chi_f|J_\nu|\chi_i >^* \tag{1}$$

is Hermitian, positive semi-definite. Therefore,

$$\det|W_{\mu,\nu}| \geq 0 \tag{2}$$

which, as we will see, is the same as expressing

$$\sigma_L, \sigma_T \geq 0 \tag{3}$$

If we take the response functions $(\sigma_L, \sigma_T, \sigma_{LT}, \sigma_{TT})$ defined from Dieter Drechsel, Lothar Tiator; J. Phys. G: 18 (1992) we get the (unpolarized) virtual photon differential cross section

$$\frac{d\sigma}{d\Omega} = \frac{|k|}{K_\gamma^{CM}}[\frac{W_{xx}+W_{yy}}{2} + \epsilon_L W_{zz} - \sqrt{2\epsilon(1+\epsilon)}\mathrm{Re}(W_{xz}) + \epsilon\frac{W_{xx}-W_{yy}}{2})] \tag{4}$$

insert the response functions and we get

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_T}{d\Omega} + \epsilon\frac{d\sigma_L}{d\Omega} + \sqrt{2\epsilon(1+\epsilon)}\frac{d\sigma_{LT}}{d\Omega}\cos\Phi + \epsilon\frac{d\sigma_{TT}}{d\Omega}\cos^2\Phi \tag{5}$$

therefore we have these four relations

$$\frac{d\sigma_T}{d\Omega} = \frac{W_{xx}+W_{yy}}{2} \tag{6}$$

$$\frac{d\sigma_L}{d\Omega} = \epsilon_L W_{zz} \tag{7}$$

$$\frac{d\sigma_{LT}}{d\Omega}\cos\Phi = -\mathrm{Re}(W_{xz}) \tag{8}$$

$$\frac{d\sigma_{TT}}{d\Omega}\cos^2\Phi = \frac{W_{xx}-W_{yy}}{2} \tag{9}$$

the $\frac{|k|}{K_\gamma^{CM}}$ term is always positive so it can be dropped because of the positive determinant. Using the Cauchy-Schwarz inequality, this determinant also implies that

$$W_{ii}W_{jj} \geq |W_{ij}|^2 \tag{10}$$

which we can view as

$$W_{xx}W_{zz} \geq |W_{xz}|^2 \tag{11}$$

by plugging this into our definition for $\frac{d\sigma_{LT}}{d\Omega}$ we have this inequality

$$|\frac{d\sigma_{LT}}{d\Omega}| = |W_{xz}| \leq \sqrt{W_{xx} \cdot W_{zz}} \tag{12}$$

similarly, we can use the trivial relation (at maximum bound, $\Phi = 0, 2\pi$)

$$2|\frac{d\sigma_T}{d\Omega}| = W_{xx} + W_{yy} \geq W_{xx} \tag{13}$$

again $W_{yy} \geq 0$ by the determinant. We can also see

$$|\frac{d\sigma_L}{d\Omega}| = W_{zz} \tag{14}$$

therefore taking eqn. 8, plugging in eqns. 10 & 11, and reducing

$$|\frac{d\sigma_{LT}}{d\Omega}| \leq \sqrt{|\frac{d\sigma_T}{d\Omega}||\frac{d\sigma_L}{d\Omega}|} \tag{15}$$

To find inequality for $\frac{d\sigma_{TT}}{d\Omega}$ we start with eqn. 10 reduced, plugging in eqn. 7, and reducing

$$W_{xx} - W_{yy} \leq W_{xx} + W_{yy} \tag{16}$$

since $W_{xx} - W_{yy} \leq W_{xx}$. We can plug in eqn. 7 again and eqn. 9 now

$$|\frac{d\sigma_{TT}}{d\Omega}| \leq |\frac{d\sigma_T}{d\Omega}| \tag{17}$$

we can rearrange eqns. 15 and 16 (subbing $|\frac{d\sigma_{ab}}{d\Omega}| \equiv \sigma_{ab}$)

$$\rho_{LT} \equiv \frac{\sigma_{LT}}{\sqrt{\sigma_T \sigma_L}} \leq 1.0, \tag{18}$$

where ρ_{LT} quantifies the interference strength between longitudinal and transverse amplitudes (via the W_{xz} component of the hadronic tensor).

$$\rho_{TT} \equiv \frac{\sigma_{TT}}{\sigma_T} \leq 1.0, \tag{19}$$

where ρ_{TT} characterizes the azimuthal angular dependence due to interference between orthogonal transverse photon polarizations.

This gives us the Cauchy-Schwarz virtual photon cross section

$$\sigma = \sigma_T + \epsilon\sigma_L + \sqrt{2\epsilon(1+\epsilon)} \cdot \rho_{LT} \cdot \sqrt{\sigma_L \sigma_T} \cos\Phi + \epsilon \cdot \rho_{TT} \cdot \sigma_T \cos^2\Phi \tag{20}$$