Asymmetry Errors

→ Need to modify formula in case of non-negligible subtractions

$$A_{LU} = \frac{1}{P}\,\frac{Y^+ - Y^-}{Y^+ + Y^-} \qquad Y^{\pm} = Y_D^{\pm} - Y_R^{\pm} - Y_P^{\pm}$$

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Define $N^{\pm} = Y_D^{\pm} + Y_R^{\pm} + Y_P^{\pm}$

Short derivation

$$\delta A^2 = \left(\frac{\partial A}{\partial Y^+}\right)^2 (\delta Y^+)^2 + \left(\frac{\partial A}{\partial Y^-}\right)^2 (\delta Y^-)^2$$

where $(\delta Y^{\pm})^2 = (\delta Y_D^{\pm})^2 + (\delta Y_R^{\pm})^2 + (\delta Y_P^{\pm})^2$

Counting statistics : $\delta Y_D^{\pm} = \sqrt{Y_D^{\pm}}$, so

$$(\delta Y^{\pm})^2 = Y_D^{\pm} + Y_R^{\pm} + Y_P^{\pm}$$
$$= N^{\pm}$$

$$\frac{\partial A}{\partial Y^+} = \frac{(Y^+ + Y^-)\cdot(1) - (Y^+ - Y^-)\cdot 1}{P(Y^+ + Y^-)^2} = \frac{2}{P}\,\frac{Y^-}{(Y^+ + Y^-)^2}$$

$$\frac{\partial A}{\partial Y^-} = \frac{(Y^+ + Y^-)\cdot(-1) - (Y^+ - Y^-)\cdot 1}{P(Y^+ + Y^-)^2} = \frac{-2}{P}\,\frac{Y^+}{(Y^+ + Y^-)^2}$$

Then : $\delta A^2 = \left(\frac{\partial A}{\partial Y^+}\right)^2 (\delta Y^+)^2 + \left(\frac{\partial A}{\partial Y^-}\right)^2 (\delta Y^-)^2$

$$= \frac{4}{P^2}\left(\frac{(Y^+)^2 N^- + (Y^-)^2 N^+}{(Y^+ + Y^-)^4}\right)$$

$$\Rightarrow \quad \delta A = \frac{2}{P(Y^+ + Y^-)^2}\sqrt{(Y^+)^2 N^- + (Y^-)^2 N^+}$$

Long Derivation (Explicitly term-by-term)

$$\delta A^2 = \sum_{a=\pm} \sum_{b=D,R,P} \left(\frac{\partial A}{\partial Y_b^a}\right)^2 (\underbrace{\delta Y_b^a}_{=Y_b^a})^2 \longrightarrow \text{6 total derivatives}$$

$$A = \frac{1}{p}\left(\frac{(Y_D^+ - Y_R^+ - Y_P^+) - (Y_D^- - Y_R^- - Y_P^-)}{Y_D^+ - Y_R^+ - Y_P^+ + Y_D^- - Y_R^- - Y_P^-}\right)$$

$$= \frac{1}{p}\left(\frac{Y_D^+ - Y_R^+ - Y_P^+ - Y_D^- + Y_R^- + Y_P^-}{Y_D^+ - Y_R^+ - Y_P^+ + Y_D^- - Y_R^- - Y_P^-}\right) \equiv \frac{1}{p}\frac{n}{d}$$

Derivatives will look very similar for each term. 4 distinct cases based on sign of term in numerator n and denominator d:

	n	d
Y_D^+	$+$	$+$
Y_D^-	$-$	$+$
Y_R^+	$-$	$-$
Y_R^-	$+$	$-$
Y_P^+	$-$	$-$
Y_P^-	$+$	$-$

$$\frac{\partial A}{\partial Y_D^+} = \left(\frac{d(+1) - n(+1)}{p\,d^2}\right) = \frac{1}{p}\left(\frac{d-n}{d^2}\right)$$

$$\frac{\partial A}{\partial Y_D^-} = \left(\frac{d(-1) - n(+1)}{p\,d^2}\right) = -\frac{1}{p}\left(\frac{d+n}{d^2}\right)$$

$$\frac{\partial A}{\partial Y_R^+} = \frac{\partial A}{\partial Y_P^+} = \left(\frac{d(-1) - n(-1)}{p\,d^2}\right) = \frac{1}{p}\left(\frac{n-d}{d^2}\right)$$

$$\frac{\partial A}{\partial Y_R^-} = \frac{\partial A}{\partial Y_P^-} = \left(\frac{d(+1) - n(-1)}{p\,d^2}\right) = \frac{1}{p}\left(\frac{d+n}{d^2}\right)$$

$$n+d = Y_D^+ - Y_R^+ - Y_P^+ - Y_D^- + Y_R^- + Y_P^- + Y_D^+ - Y_R^+ - Y_P^+ + Y_D^- - Y_R^- - Y_P^-$$
$$= 2(Y_D^+ - Y_R^+ - Y_P^+) = 2Y^+$$

$$n-d = Y_D^+ - Y_R^+ - Y_P^+ - Y_D^- + Y_R^- + Y_P^- - Y_D^+ + Y_R^+ + Y_P^+ - Y_D^- + Y_R^- + Y_P^-$$
$$= 2(-Y_D^- + Y_R^- + Y_P^-) = -2Y^-$$

$$\frac{\pm 1}{p}\left(\frac{n+d}{d^2}\right) = \frac{\pm 2}{p}\frac{Y^+}{(Y^+ + Y^-)^2} \qquad \frac{\pm 1}{p}\left(\frac{d-n}{d^2}\right) = \frac{\pm 2}{p}\frac{Y^-}{(Y^+ + Y^-)^2}$$

Then $\delta A^2 = \sum_{a=\pm} \sum_{\substack{b=D,\\ R,P}} \left(\frac{\partial A}{\partial Y_b^a}\right)^2 (\delta Y_b^a)^2$

$$= \left(\frac{\partial A}{\partial Y_D^+}\right)^2 Y_D^+ + \left(\frac{\partial A}{\partial Y_R^+}\right)^2 Y_R^+ + \left(\frac{\partial A}{\partial Y_P^+}\right)^2 Y_P^+ +$$
$$\left(\frac{\partial A}{\partial Y_D^-}\right)^2 Y + \left(\frac{\partial A}{\partial Y_R^-}\right)^2 Y_R^- + \left(\frac{\partial A}{\partial Y_P^-}\right)^2 Y_P^-$$

$$= \frac{4Y^{-2}}{p^2 d^4} Y_D^+ + \frac{4Y^{-2}}{P^2 d^4} Y_R^+ + \frac{4Y^{-2}}{p^2 d^4} Y_P^+ + \frac{4Y^{+2}}{P^2 d^4} Y_D^-$$
$$+ \frac{4Y^{+2}}{p^2 d^4} Y_R^- + \frac{4Y^{+2}}{p^2 d^4} Y_P^-$$

$$= \left(\frac{4}{p^2}\right)\left(\frac{(Y^+)^2(Y_D^- + Y_R^- + Y_P^-) + (Y^-)^2(Y_D^+ + Y_R^+ + Y_P^+)}{(Y^+ + Y^-)^4}\right)$$

$$\Rightarrow \delta A = \frac{2}{P(Y^+ + Y^-)^2} \sqrt{(Y^+)^2 N^- + (Y^-)^2 N^+}$$

Note: Recovering the previous formula

If subtractions are negligible, then $Y^\pm \approx N^\pm$, and

$$\delta A^2 = \frac{4}{p^2}\left(\frac{(Y^+)^2 Y^- + (Y^-)^2 Y^+}{(Y^+ + Y^-)^4}\right)$$
$$= \frac{4}{p^2}\left(\frac{Y^+ Y^- (Y^+ + Y^-)}{(Y^+ + Y^-)^4}\right)$$
$$= \frac{4}{p^2} \frac{Y^+ Y^-}{(Y^+ + Y^-)^3}$$